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This book is designed to prepare students for taking the SAT I--Reasoning Test using a problem-solving focus. It clarifies the purpose, structure, and use of the SAT I and provides meaningful instructional material and explanations which illuminate the necessary skills. The book begins with a review of essential fundamentals, including useful concepts from number theory, arithmetic, algebra, and geometry. This is followed by 30 model solutions of typical multiple-choice problems and 10 model solutions of carefully selected quantitative comparisons. Next, there are 22 practice tests, followed by a section of complete solutions to all of the problems. Students are urged to read the solutions presented and compare them with their own solutions in an effort to find the best method of solution for each type of problem. A cross-index is included to enable the student to locate particular topics within the instructional material. (MKR)

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Problem-Solving Strategies and Practice Tests

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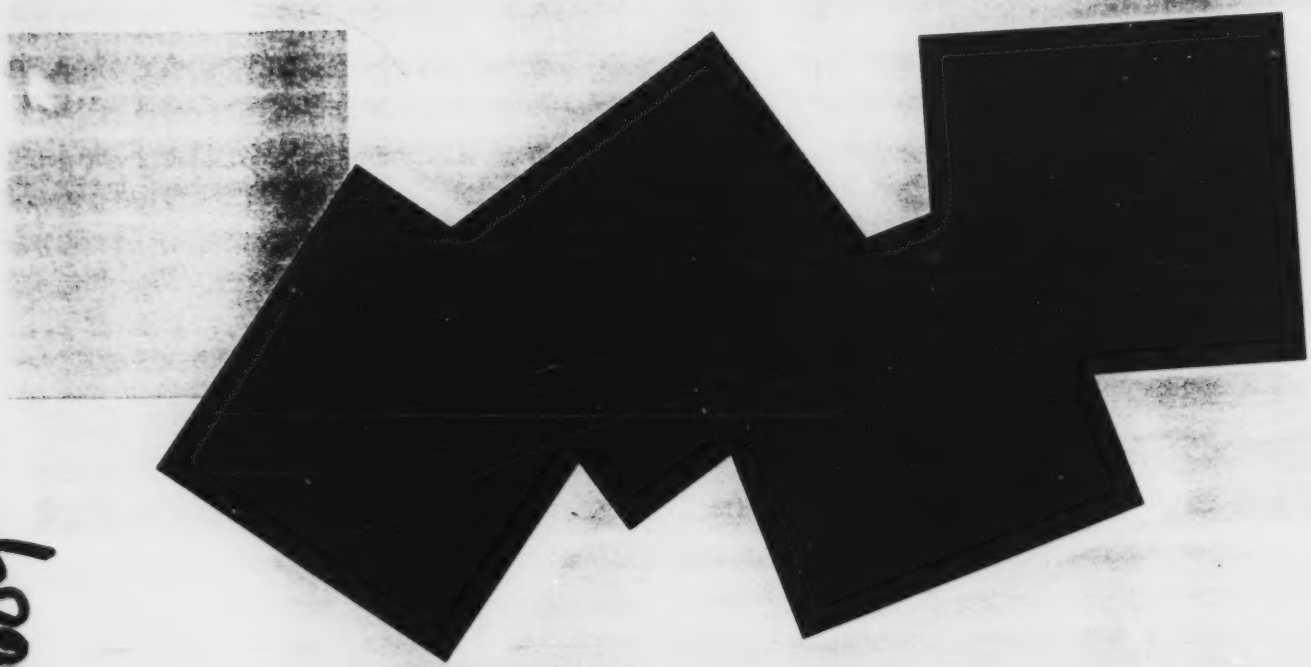

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Students! Get Ready
for the
Mathematics for SAT*I

Students! Get Ready for the Mathematics for SAT*I

Problem-Solving Strategies and Practice Tests

Alfred S. Posamentier

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Preface

The *SAT I - Reasoning Test* used by more than 2,000 American colleges as a screening device to select students for admission. These colleges examine many pieces of information about each applicant; the *SAT I* is one important yardstick they use to measure ability for college work. This book has a single purpose: to help college applicants prepare for this crucial test. Every page of discussion, every bit of review, every practice question is designed to lead the student toward the highest score on this test that he or she is capable of attaining. No time or space is wasted on material that does not relate directly to preparation for the *SAT I*.

To the extent permitted by the tight security surrounding the *SAT I*, the sample questions and practice tests are as close to the "real thing" as they could possibly be. Students who use the material in this book can be confident that they are simulating an actual test situation and meeting the same kinds of problems (perhaps even a bit more challenging!) that they will face when they take the *SAT I*.

There are no short cuts to successful *SAT I* preparation. Reasoning skills do not grow overnight from "cram" sessions, memory tricks, or mental gimmicks. The author is convinced, however, that many students fail to demonstrate the abilities they do have because of ignorance of the nature of the *SAT I*, their limited work with problems requiring the *SAT I* type of reasoning, and their confusion about the techniques they should use. This book will help them in two ways:

- It will clarify the purpose, structure, and use of the *SAT I*.
- It will provide meaningful instructional material and explanations which illuminate the necessary skills.

Used conscientiously, this book will give the student a thorough working knowledge of the *SAT I* and the confidence which comes from systematic practice, increased awareness of the nature of the task, and having the tools to do the job right.

Specific preparation for the Mathematics Section of the *SAT I* begins with the review of essential fundamentals, including useful concepts from number theory, arithmetic, algebra, and geometry. This is followed by 30 model solutions of typical multiple-choice problems and 10 model solutions of carefully selected quantitative comparisons. Next, there are 22 practice tests, followed by a section of complete solutions to all of the problems. Students are urged to read the solutions presented and compare them with their own solutions in an effort to find the best method of solution for each type of problem. A cross-index is included to enable the student to locate particular topics within the instructional material.

The value of this book is in its clear presentation of typical *SAT I* problems and the elucidation of the reasoning and skills required. The discussions show the pitfalls that may be present and light the way to logical solutions. The effectiveness of these analyses was made possible by the enthusiastic cooperation of many hundreds of students who took the practice tests, asked questions, challenged, groped, thought - and learned - as well as the many high school teachers who read the manuscript and made valuable suggestions. The author wishes to express his gratitude to these students and teachers who enthusiastically offered advice when asked.. A special word of thanks is also due to Naomi Feiner, Peter Lau, Joyce Dato, Roland Franzke, and Barbara Lowin, whose typing and retyping, checking and rechecking of manuscript and proofs were of immeasurable value in bringing this work to fruition.

Alfred S. Posamentier
February, 1996

ABOUT THE AUTHOR

Alfred S. Posamentier is Professor of Mathematics Education and Deputy Dean of the School of Education of the City College of the City University of New York. He is the author and coauthor of numerous mathematics books for teachers and secondary school students. As a guest lecturer, he favors topics regarding aspects of mathematics problem solving and the introduction of uncommon topics into the secondary school realm for the purpose of enriching the mathematics experience of those students. The development of this book reflects these penchants.

After completing his A.B. degree in mathematics at Hunter College of the City University of New York, he took a position as a teacher of mathematics at Theodore Roosevelt High School in the Bronx (New York), where he focused his attention on improving the students' problem-solving skills. He also developed the school's first mathematics teams (both at the junior and senior level) and established a special class whose primary focus was methods of mathematics problem solving.

Immediately upon joining the faculty of the City College (after having received his masters' degree there), he began to develop inservice courses for secondary school mathematics teachers on the topic of mathematics problem solving.

Dr. Posamentier received his Ph.D. from Fordham University (New York) in mathematics education and since has extended his reputation to Europe. He is an **Honorary Fellow** at the South Bank University (London, England). He has been visiting professor at several Austrian, British, and German Universities, and most recently at the University of Vienna and at the Technical University of Vienna. At the former, he was a Fulbright Professor.

In recognition of his outstanding teaching, he was recently named **Educator of the Year** by the City College Alumni Association and had a day named in his honor by the City Council President of New York City. More recently, he was awarded the **National Medal of Honor** from the Austrian government.

Naturally, with his penchant for mathematics problem solving, he has been very concerned that students have a proper introduction to, and solid preparation for, problem solving strategies. This concern motivated the development of this book.

INTRODUCTION

The ***SAT I - Reasoning Test*** examines your potential for college work. It is clearly not the only indicator of your potential. Your high school grades probably predict best how well you will do in college. However, college admissions officers also rely on the yardstick of standardized tests when making admission decisions because such tests compare students from many different schools and regions. The *SAT I* is one such important yardstick. Your scores on the *SAT I*, expressed within a range of 200 to 800 and as a percentile rank, allow comparison with other college-bound students.

It would be impractical in a 3-hour exam to test you in every subject area covered by your high school courses. Rather, the *SAT I* is written in a way that tests your verbal and mathematical reasoning ability to apply knowledge gained from these courses. The approach you take to questions posed on the *SAT I* indicates the level of reasoning you have reached by the end of your high school education. The tests of verbal and mathematical reasoning are designed to reflect how developed your reading comprehension and problem-solving skills are and, consequently, how successful you will be in college.

Format of the *SAT I*

There are three test sections of verbal reasoning and three test sections of mathematical reasoning on the *SAT I*, separated into sections as follows:

Two 30-minute sections and one 15-minute section of **verbal reasoning ability** (with an emphasis on critical reading questions), comprising 78 questions in all.

Two 30-minute sections and one 15-minute section on **mathematical reasoning ability**, comprising 60 questions in all. The questions are of three types:

- **35 multiple-choice** questions, each having 5 possible choices
- **15 quantitative comparison** questions, also multiple choice, each having 4 possible choices
- **10 student-produced response** questions, answered on a grid by blackening spaces with the solution determined by the test taker

There is a also a section on either verbal or mathematics skills that is not counted in the examinee's score; it is used to insure that *SAT I* scores remain consistent from test to test, and it "tries out" questions that may appear on future tests. The scores from the verbal and mathematics sections of the test are reported separately, so one score does not affect the other.

A final point about the format of the *SAT I*: Because it is a timed test, it is essential that you read directions quickly but carefully, and that you judge rapidly as you go along which questions you can answer easily and which will require more time. Naturally, you should first answer the questions whose answers are more obvious to you, then proceed to the more difficult questions. It

is very frustrating to have questions left unread when time runs out, as you would no doubt have been able to answer some of these with little effort.

You will have a great advantage if you understand the directions before you arrive at the test site. If you have not already done so, you should obtain the free publications of the College Board, *Taking the SAT I - Reasoning Test* and *Registration Bulletin*. These will give you up-to-date information on fees, locations, test dates, and score reporting. If your high school does not have these publications, you can get them by writing to:

The College Board SAT Program
P.O. Box 6200
Princeton, NJ 08541-6200

You can also find out from the College Board how to obtain previously administered *SAT I* tests, if they are not available from your high school.

The Mathematics Section of the *SAT I*

In your high school mathematics classes you have learned the rudiments of algebra and geometry; perhaps you have studied further into other areas. The *SAT I* tests only as far as the information found in the basic courses. At the same time, it tests other aspects of mathematical ability, namely reasoning and problem solving. Doing well in these areas requires excellent methods of data collection, pattern detection, deductive reasoning, and many other intellectual strategies. Such strategies are not instantly acquired as you learn specific mathematical processes (such as solving a linear equation or finding the length of the third side of a right triangle). Rather, they are developed over a long period of time.

Students who receive methodical instruction in reasoning and problem solving exhibit the strongest mathematical skills. In addition, those who have taken a full sequence of mathematics courses tend to have sharper skills than students who have stopped after basic mathematics. As might be expected, the better trained group tends to score higher on the *SAT I*.

What about those of you in the other group, whose preparation in problem solving may be weak? Are you able to achieve higher scores on the *SAT I*? If you have learned your basic mathematics well and are willing to study hard for the test, the answer is a definite *"Yes!"* However, you must also learn *how* and *what* to study. It is true that mathematical ability is a quality that is largely developed as part of your general academic ability; but regardless of how much "inborn talent" you believe you have, or the level of mathematics study you have reached, your *SAT I* scores can still fail to be a true measure of your ability. Too often, students achieve scores lower than they are capable of achieving because they do not prepare properly. If you follow the plan in this book, you can help to ensure that your *SAT I* scores will be as high as possible.

Using This Book

The book's problem-solving techniques and sample problems are the keys that unlock the doors to understanding the nontraditional questions found on the *SAT I*. As do those on the *SAT I*, the

questions in this book often require you to see underlying patterns and relationships in the given mathematical data; intermediate steps must usually be performed before you can arrive at a correct answer. Many examples and questions in this book are intentionally more difficult than those you would find on a typical *SAT I*. This should not discourage you; the sample problems offer plenty of guidance to answering more challenging questions. You will find actual *SAT I* questions far easier if you master those presented here, and your anxiety will lessen considerably.

Thus, by revealing the *strategies* you will need to intelligently approach and answer *SAT I* mathematics questions, this book goes beyond simply removing the elements of surprise and lack of familiarity with certain types of questions that students generally experience when taking the test. This kind of preparation makes it possible for you to *get the score you should get*, based on what you know of the topics covered. The College Board says that you need to know the following topics:

Arithmetic - simple addition, subtraction, multiplication, and division; percent; data interpretation (including mean, median, and mode); odd and even numbers; prime numbers; divisibility

Algebra - negative numbers; substitution; simplifying algebraic expressions; simple factoring; linear equations; inequalities; simple quadratic equations; positive integer exponents; roots of numbers; sequences

Geometry - area and perimeter of a polygon; area and circumference of a circle; volume of a box, cube, and cylinder; Pythagorean Theorem and special properties of isosceles, equilateral, and right triangles; 30-60-90 and 45-45-90 triangles; properties of parallel and perpendicular lines; simple coordinate geometry; slope; similarity; geometric visualization

Other - logical reasoning; newly defined symbols and operations that are based on commonly used symbols and operations; probability and counting

The following sections make up this book:

A discussion on gaining the best advantage from *using a calculator* on the *SAT I*

A brief *review of the essentials* of arithmetic, algebra, probability, and geometry

A *description and explanation of the format* of the multiple-choice questions, the quantitative comparison questions, and the student-produced response questions. The College Board has given permission to use the exact test instructions in this book as they appear on the *SAT I* (e.g., a sample grid for the student response questions is included along with important points to be aware of when filling in the grid)

Sample problems from each area of mathematics covered on the test, with detailed solutions that highlight somewhat unusual problem-solving strategies, typically not covered in the regular school curriculum

A discussion of *quantitative comparision questions*, which ask you to compare two quantities

Practice assessment tests with numerous questions representative of the *SAT I*. A time limit is given and questions are arranged in ascending order of difficulty, just as they are on an actual *SAT I*.

Detailed solutions to the problems on the practice assessment tests

Subject cross-index to enable the reader to quickly find problems in specific areas

An *answer key* for a quick check of the answers arrived at during practice

There is another very important advantage to be gained from using this book: From it you will get your first insight into *how* you need to study in order to succeed in college. Intense and systematic study for the *SAT I* with the aid of this book will help you develop an approach to thinking skills and problem solving that advanced subjects demand.

Calculator Use on the *SAT I - Reasoning Test*

You might think that the advantages of using a calculator on the *SAT I* would be great. In fact, the main advantage afforded is that you can avoid arithmetic errors, which would otherwise lower your score, in spite of your good problem-solving skills.

There are also negative aspects to optional calculator use on the *SAT I*. Test writers have to reduce the number of questions whose solutions you could find by simply punching data into a calculator. Naturally, these kinds of questions are replaced with others that may not be as straightforward, but are a truer test of mathematical reasoning. For you, the test taker, other problems arise if you

Are unsure of how to use calculator keys for signed numbers, fractions, percents, and exponents

Are not adept at handling fractions that convert to repeating decimals

Cannot quickly change any decimal number (repeating or terminating) back to a fraction to match the multiple-choice answers

Are simply careless when hitting buttons

Recognizing these pitfalls, this book will illustrate, in specific examples, how using a calculator can affect the form of your answer and whether a problem can be solved better with or without a

calculator. In addition, the College Board publication, *Taking the SAT I - Reasoning Test 1994-1995* offers some excellent points and cautions regarding calculator use:

> *All questions on the test can be answered without a calculator.*
>
> No test questions will require messy or tedious calculations.
>
> Don't use a calculator on every question. Demonstrate good judgment in determining when and when not to use the calculator.
>
> Calculators with sophisticated functions are unlikely to provide an advantage.
>
> *Using your calculator too much can cost you time.* [The calculator] is meant to aid you in problem solving, not to get in the way.
>
> Think through how you will solve each problem, then decide whether to use the calculator.

The College Board also reports the fact that students who use calculators "do slightly better than [those] who do not." This book can show you how to maximize the benefits derived from calculator use and attain results that are more impressive than just "slightly better."

Student-Produced Response Questions

Another recent change in the format of the *SAT I* is the inclusion of questions for which you write your own answers. With these, you have a chance to work a problem without being constrained or influenced by multiple choice answers. Instead, you write your answers on a grid by blackening spaces. You will probably welcome this feature of the *SAT I* as you are no doubt used to taking mathematics tests that are "open-ended," tests that do not offer a choice of responses.

Two tests of open-ended questions are included in this book. In this section of the book, the actual answer grid is duplicated, and certain points you should be aware of when filling in the grid are discussed. (For example, you can often choose between writing the fraction or decimal form of an answer in the grid, but care in rounding is essential.)

One reason for the addition of student-produced response questions to the *SAT I* was to make the test fairer. If you are the kind of test taker who is most comfortable with this format, seizing the opportunity to answer these open-ended questions correctly can significantly improve your score.

Although there are only 10 student-produced response questions on the *SAT I*, the psychological advantage they can provide is great indeed: Because you are unrestricted by a choice of solutions that may not immediately resemble your own, the panic you may have otherwise experienced is reduced on this section. This allows you to relax and answer this set of questions with a clearer mind.

I. Review of Mathematical Principles

All SAT I problems can be solved using simple mathematical operations, but your score will be greatly affected by your familiarity with these operations and by the speed and accuracy with which you carry them out. It will do you little good to know how to work a problem if you then get it wrong by making an error in reducing a fraction or converting it to percent form. Therefore, an important part of preparing for the SAT I is brushing up on your fundamental mathematical skills.

This review is intended merely as a refresher of concepts you should have acquired from your previous study of mathematics. Read through these pages carefully, making sure you are familiar with and understand each of the principles and techniques illustrated. If there are any that you feel unsure of, don't take any chances; get a math text or review book and study the corresponding topics until you have mastered them.

Note particularly the rules for divisibility given on pages 6-8. This information is seldom encountered in ordinary textbooks, so it may be new to you. It can be an important time-saver in arithmetic computations and is therefore well worth learning.

RULES FOR DIVISIBILITY

It is often very useful to be able to determine what numbers will divide a given number exactly. For example, to reduce the fraction $\frac{2937}{4521}$ to lowest terms, we have to find common divisors for numerator and denominator. To do this by trial and error is a very time-consuming procedure. By using the following rules for divisibility, we may reduce the amount of work considerably.

Rule for divisibility by 2
The last digit of the given number must be even (or 0).

Example
1838 is divisible by 2, since the last digit, 8, is even.

Rule for divisibility by 3
The sum of the digits of the given number must be divisible by 3.

Example
2937 is divisible by 3, since the sum of the digits, $2 + 9 + 3 + 7 = 21$, is divisible by 3.

Rule for divisibility by 4
The last two digits of the given number must form a number that is divisible by 4.

Example
7312 is divisible by 4, since the last two digits form the number 12, which is divisible by 4.

Rule for divisibility by 5
The last digit of the given number must be either 0 or 5.

Example
3785 is divisible by 5, since the last digit is 5.

Rule for divisibility by 6

The rules for divisibility by both 2 *and* 3 must hold, since their product is 6.

Example

1758 is divisible by 2, since the last digit, 8, is even.

AND

1758 is divisible by 3, since the sum of the digits, 1 + 7 + 5 + 8 = 21, is a number divisible by 3.

Therefore, 1758 is divisible by 6.

Rule for divisibility by 7

Cut off the last digit from the given number; then subtract twice this deleted digit from the remaining number. The result obtained must be divisible by 7. For large numbers, this procedure may have to be repeated.

Example 1

959 is divisible by 7:

	95 | 9
(Subtract 2 x 9)	18
	77 (which is divisible by 7)

Example 2

5803 is divisible by 7:

	580 | 3
(Subtract 2 x 3)	6
	574
(Repeat process)	57 | 4
(Subtract 2 x 4)	8
	49 (which is divisible by 7)

Rule for divisibility by 8

The last three digits of the given number must form a number that is divisible by 8.

Example

57256 is divisible by 8, since the number formed by the last three digits, 256, is divisible by 8.

Rule for divisibility by 9

The sum of the digits of the given number must be divisible by 9.

Example

5382 is divisible by 9, since the sum of the digits, 5 + 3 + 8 + 2 = 18, is divisible by 9.

Rule for divisibility by 10

The last digit of the given number must be 0.

Example

1970 is divisible by 10, since the last digit is 0.

Rule for divisibility by 11

The difference of the two sums of alternate digits of the given number must be divisible by 11.

Example 1

91927 is divisible by 11, since the difference of the sums of alternate digits, (9 + 9 + 7) - (1 + 2) = 22, is divisible by 11.

Example 2

4521 is divisible by 11, since the difference of the sums of the alternate digits, (4 + 2)-(5 + 1) = 0, is divisible by 11. (Note: Zero is divisible by any nonzero number.)

Rule for divisibility by 12

The rules for divisibility by both 3 *and* 4 must hold, since their product is 12.

Example

1524 is divisible by 3, since 1 + 5 + 2 + 4 = 12 is a number divisible by 3.

AND

1524 is divisible by 4, since the last two digits form a number, 24, which is divisible by 4. Therefore, 1524 is divisible by 12.

Rules for divisibility by composites (i.e., nonprimes) greater than 12 follow the same scheme as that for 12 and 6. For example, in order for a given number to be divisible by 15, the rules for divisibility by 3 *and* 5 must both hold.

ORDER OF OPERATIONS

We are frequently faced with the problem of evaluating an expression like the following:

$$5 - 7(5 - 2) + 8 + 4 + 6$$

If there were no generally accepted rules for the order in which to perform the indicated operations, there would be several possible results. Mathematicians have therefore adopted the following rules for the order of operations:

1. Perform the operations indicated inside each pair of parentheses. If there are parentheses within parentheses, start with the interior ones and work outwards.

2. When all parentheses have been cleared away, perform the multiplications and divisions in order from left to right.

3. Last, perform the additions and subtractions in order from left to right.

Example

5 - 7(5 - 2) + 8 ÷ 4 + 6 = ?

By rule 1, we obtain:
5 - 7 × 3 + 8 ÷ 4 + 6

By rule 2, we obtain:
5 - 21 + 2 + 6

By rule 3, we obtain:
-8

A BRIEF REVIEW OF FRACTIONS, DECIMAL FRACTIONS, AND PERCENTS

1. Parts of a fraction

In the fraction $\frac{3}{5}$, the *numerator* is 3, and the *denominator* is 5.

2. Proper and improper fractions

If the numerator of a fraction is less than the denominator, then it is a proper fraction (e.g., $\frac{3}{4}$). If the denominator of a fraction is less than the numerator, then it is an improper fraction (e.g., $\frac{5}{3}$).

3. Mixed numbers

A mixed number is one in which a whole number and a proper fraction are combined (e.g., $5\frac{2}{3}$). In order to facilitate computation, it is often advisable to change a mixed number to an improper fraction.

Example

To change $5\frac{2}{3}$ to an improper fraction, multiply 5 by 3 and add 2 to the product to get the new numerator, and use the original denominator (3). In this way, $5\frac{2}{3} = \frac{17}{3}$.

4. Reducing fractions to lowest terms

To reduce a fraction to lowest terms, divide both numerator and denominator by a common factor (where possible). This procedure is to be repeated until no common factor exists. To facilitate the search for a common factor, see the section on *"Rules for Divisibility,"* pages 6-8.

Example

To reduce $\frac{2937}{4521}$ to lowest terms, divide both numerator and denominator by 3 and 11 (or by their product, 33) to get $\frac{89}{137}$.

5. Addition of fractions

In order to add fractions, we must change each of the fractions to equivalent fractions with the same denominator (preferably the lowest common denominator). Add the numerators (to get the numerator of the sum).

Example

$$\frac{2}{3}+\frac{5}{8}+\frac{7}{12}=?$$

Since the lowest common denominator of the three denominators (3, 8, and 12) is 24, we can change each fraction to an equivalent fraction with a denominator of 24:

$$\frac{2}{3}=\frac{2}{3}\cdot\frac{8}{8}=\frac{16}{24}$$
$$\frac{5}{8}=\frac{5}{8}\cdot\frac{3}{3}=\frac{15}{24}$$
$$\frac{7}{12}=\frac{7}{12}\cdot\frac{2}{2}=\frac{14}{24}$$

By addition, we get: $\frac{16+15+14}{24}=\frac{45}{24}=\frac{15}{8}=1\frac{7}{8}$

6. Subtraction of fractions

In order to subtract one fraction from another, we must change each of the fractions to equivalent fractions with a common denominator (as in addition). We must then subtract the numerators (keeping the common denominator).

Example

Subtract $\frac{2}{3}$ from $\frac{4}{5}$.

Since the lowest common denominator of the two denominators (3 and 5) is 15, we must change each fraction to an equivalent fraction with denominator of 15:

$$\frac{4}{5}=\frac{4}{5}\cdot\frac{3}{3}=\frac{12}{15}$$
$$\frac{2}{3}=\frac{2}{3}\cdot\frac{5}{5}=\frac{10}{15}$$

Subtracting the numerators, we get:

$$\frac{12-10}{15}=\frac{2}{15}$$

7. Multiplication of fractions

To multiply fractions, multiply the numerators to get the numerator of the product, and multiply the denominators to get the denominator of the product. To simplify the operation, divide out any common factors between numerators and denominators before doing the multiplications

Example

Multiply $\frac{15}{22}$ by $\frac{33}{65}$.

Divide numerator and denominator by 5, the common factor of 15 and 65:

$$\frac{\cancel{15}^{\,3}}{22}\cdot\frac{33}{\cancel{65}_{\,13}}=\frac{3}{22}\cdot\frac{33}{13}$$

Divide numerator and denominator by 11, the common factor of 22 and 33:

$$\frac{3}{\cancel{22}_{\,2}}\cdot\frac{\cancel{33}^{\,3}}{13}=\frac{3}{2}\cdot\frac{3}{13}=\frac{9}{26}$$

8. Division of fractions

To divide one fraction by another, invert the divisor and then multiply the dividend by the inverted divisor.

Example

Divide $\frac{2}{3}$ by $\frac{6}{7}$.

$$\frac{2}{3}+\frac{6}{7}=\frac{\cancel{2}^{\,1}}{3}\cdot\frac{7}{\cancel{6}_{\,3}}=\frac{7}{9}$$

9. Comparing fractions

See Sample Problem 9, page 45.

10. Addition of decimal fractions

The decimal fractions to be added must be placed in a column so that all the decimal points are "lined up."

Example

.007 + 7.3 + 56 + 10.502 = ?

Write the numbers to be added in a column:

```
  .007
 7.3
56.
10.502
73.809
```

11. Subtraction of decimal fractions

The decimal fractions to be subtracted must be placed with the decimal points "lined up."

Example

Subtract 5.2 from 2.05.

```
 5.20
-2.05
 3.15
```

12. Multiplication of decimal fractions

Multiply the numbers without regard to the decimal points. Then place the decimal point in the product so that there are as many digits to the right of it as the total number of digits to the right of the decimal point in the original numbers. A final zero in the product counts as a digit.

Example

Multiply 5.805 by 3.02.

```
  5.805   (3 digits to the right of the decimal point)
  3.02    (2 digits to the right of the decimal point)
 11610
17415
17.53110  (5 digits to the right of the decimal point)
```

13. Division of decimal fractions

(1) Move decimal point to right of final digit of divisor. (2) Move decimal point as many places to right in dividend as it was moved in divisor. (Add zeros, if necessary, to dividend.) (3) Divide by long division. (4) Place decimal point in quotient directly above displaced decimal point in dividend.

Example

Divide 12.243 by 2.31.

```
          (4)
           v
          5.3
      ----------
2.31. | 12.24.3
      |  11 55
(1)   |    69 3   (2)
      |
      |    69 3
```

14. Converting a fraction to decimal form

Divide the numerator by the denominator.

Example

To convert $\frac{1}{8}$ to decimal form, divide 1.000 by 8 to get .125.

15. Repeating decimals

Fractions convert either to terminating decimals, as in rule 14 (above), or to repeating decimals.

Examples

$\frac{1}{3} = 0.33\ldots$, often written as $0.\overline{3}$

$\frac{1}{11} = 0.0909\ldots$ or $0.\overline{09}$

$\frac{611}{4950} = 0.12343434\ldots$ or $0.12\overline{34}$

16. Converting a repeating decimal to a fraction

Determine the fraction represented by the repeating decimal by following the procedure in the examples below:

0.333 . . . has *one* repeating digit; let $x = 0.33\ldots$;
multiply x by 10 *once* to get $10x = 3.333\ldots$; set up the subtraction:

$$\begin{array}{rcl} 10x & = & 3.33\ldots \\ -\ x & = & 0.33\ldots \\ \hline 9x & = & 3 \end{array}$$

$$x = \frac{3}{9} = \frac{1}{3}$$

0.0909. . . has *two* repeating digits; let $x = 0.0909\ldots$;
multiply x by 10 *twice* to get $100x = 9.0909\ldots$; set up the subtraction:

$$\begin{array}{rcl} 100x & = & 9.0909\ldots \\ -\ x & = & 0.0909\ldots \\ \hline 99x & = & 9 \end{array}$$

$$x = \frac{9}{99} = \frac{1}{11}$$

0.123434. . . has *two* repeating decimals; let $x = 0.123434\ldots$;
multiply x by 10 *twice* to get $100x = 12.3434\ldots$; set up the subtraction:

$$\begin{array}{rcl} 100x & = & 12.343434\ldots \\ -\ x & = & 0.123434\ldots \\ \hline 99x & = & 12.22 \end{array}$$

$$x = \frac{12.22}{99} = \frac{1222}{9900} = \frac{611}{4950}$$

17. Converting a decimal fraction to percent form

Multiply by 100%; that is, move the decimal point two places to the right and add the % sign.

Example

To change .35 to percent form, move the decimal point two places to the right (.35.) to get 35%.

18. Converting a fraction to percent form

Multiply by 100%, or change the fraction to decimal form and then to percent form. (See Nos. 14 and 17.)

Example

Change $\frac{3}{8}$ to percent form.

$\frac{3}{8} \cdot 100\% = \frac{300}{8}\% = 37.5\%$ or $\frac{3}{8} = .375 = 37.5\%$

PERCENTAGE PROBLEMS

In order to simplify the following discussion, let us define the following terms:

Percentage is the result obtained when a percent is taken of a number.

Base is the number of which a percent is being taken.

Rate is the percent taken of a number (i.e., the base).

Following are the three basic types of percentage problems:

Type of Problem	*Sample Problems*	
Finding percentage: Percentage = Rate X Base	1. Find 35% of 20. **Solution**: (.35) (20)= 7	2. Find 42% of 150. **Solution**: (.42) (150) = 63
Finding the base: $\text{Base} = \frac{\text{Percentage}}{\text{Rate}}$	1. 35% of what number is 7? **Solution**: $\frac{7}{.35} = \frac{700}{35} = 20$	2. 42% of what number is 63? **Solution**: $\frac{63}{.42} = \frac{6300}{42} = 150$
Finding the rate: $\text{Rate} = \frac{\text{Percentage}}{\text{Base}}$	1. 7 is what percent of 20? **Solution**: $\frac{7}{20} = .35 = 35\%$	2. 63 is what percent of 150? **Solution**: $\frac{63}{150} = .42 = 42\%$

EQUIVALENT FRACTIONS AND PERCENTS YOU SHOULD KNOW

$\frac{1}{2} = 50\%$	$\frac{1}{3} = 33\frac{1}{3}\%$ $\frac{2}{3} = 66\frac{2}{3}\%$	$\frac{1}{4} = 25\%$ $\frac{3}{4} = 75\%$	$\frac{1}{5} = 20\%$ $\frac{2}{5} = 40\%$ $\frac{3}{5} = 60\%$ $\frac{4}{5} = 80\%$	$\frac{1}{6} = 16\frac{2}{3}\%$ $\frac{5}{6} = 83\frac{1}{3}\%$	$\frac{1}{8} = 12\frac{1}{2}\%$ $\frac{3}{8} = 37\frac{1}{2}\%$ $\frac{5}{8} = 62\frac{1}{2}\%$ $\frac{7}{8} = 87\frac{1}{2}\%$

$\frac{1}{7} = 14\frac{2}{7}\%$	$\frac{1}{9} = 11\frac{1}{9}\%$	$\frac{1}{11} = 9\frac{1}{11}\%$	$\frac{1}{12} = 8\frac{1}{3}\%$

A BRIEF REVIEW OF ESSENTIALS OF ALGEBRA

1. Addition of signed numbers

It is important to understand the difference in meaning between a plus or minus sign used to indicate the operation "add" or "subtract," and the same signs used to indicate whether a number is positive or negative. For example, the expression:

$$5 + (-7)$$

means "add the negative number -7 to the positive number 5." (Note that the plus sign is usually omitted before positive numbers.) The expression:

$$5 - 7$$

means "subtract the (positive) number 7 from the (positive) number 5." (Note again the omission of the positive signs.)

A. To add numbers that all have the same sign, add the numbers (absolute values) and keep the same sign.

Example

$(-5) + (-22) + (-7) = -(5 + 22 + 7) = -34$

B. To add numbers with different signs, find the sum of the positive numbers and the sum of the negative numbers. Then find the difference between the absolute values of those two sums. Give the result the sign of the larger of the two sums.

Example

Add: -12, -7, +15, -10, +3, +18.

Add separately the positive and negative numbers:

$(+15) + (+3) + (+18) = +(15 + 3 + 18) = +36$

$(-12) + (-7) + (-10) = -(12 + 7 + 10) = -29$

Find the difference between the absolute values, 36 and 29:

$36 - 29 = 7$

Use the sign of the larger of the two absolute values:

$+7$

2. Subtraction of signed numbers

To subtract one signed number from another, change the sign of the subtrahend (the number being subtracted) and follow the rules for addition of signed numbers.

Example 1

Subtract -58 from +35.

$+35 - (-58) = +35 + (+58) = +(35 + 58) = +93$

Example 2

Subtract +58 from +35.

$+35 - (+58) = +35 + (-58) = -(58 - 35) = -23$

Example 3

Subtract -58 from -35.

$-35 - (-58) = -35 + (+58) = +(58 - 35) = +23$

3. Multiplication of signed numbers

To multiply two signed numbers, multiply their absolute values; then,
(a) if the signs are alike, give the product a positive sign;
(b) if the signs are unlike, give the product a negative sign.

Examples

$(+3)(-5) = -15$
$(-3)(+5) = -15$
$(-3)(-5) = +15$
$(+3)(+5) = +15$

Notes

(a) When multiplying an *odd* number of negative numbers, the product is *negative*.

Example

$(+5)(-2)(-3)(+2)(-1) = -60$

(b) When multiplying an *even* number of negative numbers, the product is *positive*.

Example

$(-5)(-2)(-3)(+2)(-1) = +60$

It follows that an odd power of a negative number yields a negative result, whereas an even power of a negative number yields a positive result.

4. Division of signed numbers

When dividing a pair of signed numbers, divide their absolute values; then apply the rule of signs for multiplication.

Examples

$(+15) \div (-3) = -5$
$(-15) \div (+3) = -5$
$(-15) \div (-3) = +5$
$(+15) \div (+3) = +5$

5. Positive exponents

$$a^n = \underbrace{a \bullet a \bullet a \bullet \cdots \bullet a}_{n \text{ times}}$$

The base is a and the exponent is n. The exponent indicates the number of times the base appears as a factor in the product.

Examples

$2^1 = 2$ $\qquad 2^3 = 2 \bullet 2 \bullet 2 = 8$ $\qquad (-2)^4 = (-2) \bullet (-2) \bullet (-2) \bullet (-2) = 16$

But $\qquad -(2)^4 = -(2) \bullet (2) \bullet (2) \bullet (2) = -16$

6. Negative exponents

$$a^{-n} = \frac{1}{a^n}$$

A negative exponent indicates that a reciprocal is to be taken, using the same base and the negative of the original exponent.

Examples

$3^{-1} = \frac{1}{3}$ $\qquad$ $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

$(-1)^{-4} = \frac{1}{(-1)^4} = \frac{1}{1} = 1$ $\qquad$ $-1^{-4} = -\frac{1}{1^4} = -\frac{1}{1} = -1$

7. Zero exponent

Any number, except zero, raised to an exponent of zero is equal to 1.

$$a^0 = 1, \quad a \neq 0$$

Zero raised to an exponent of zero is undefined. (See Rule 10.)

Examples

$3^0 = 1$ $\qquad$ $(-4)^0 = 1$ $\qquad$ [0^0 is undefined]

But $-4^0 = -(4)^0 = -1$

8. Monomials (coefficients and variables)

The expression

$$ax^n$$

is a monomial. The coefficient a represents a given numerical value; the base x represents an unknown or variable.

Examples

$2x$	2 is the coefficient
$-x$	-1 is the coefficient
$\frac{y}{3}$	$\frac{1}{3}$ is the coefficient

9. Multiplying monomials

Multiply the coefficients and add the exponents of factors with the same base.

$$(ax^n)\cdot(bx^m) = abx^{n+m}$$

Examples

$(2x^3)\cdot(3x^2) = 6x^{3+2} = 6x^5$ $\qquad$ $(-5x)\cdot(x)\cdot(9x^2) = -45x^{1+1+2} = -45x^4$

$(-5x^2y^3)\cdot(3xy^2) = -15x^3y^5$

10. Dividing monomials

Divide the coefficients, and for factors with the same base, subtract the exponent of the denominator from the exponent of the numerator.

$$\frac{x^n}{x^m} = x^{n-m}$$

Recall from rule 7 that $x^0 = 1$. From the rule for dividing monomials:

$$1 = \frac{x^n}{x^n} = x^{n-n} = x^0$$

Examples

$$\frac{8x^3}{2x^2} = 4x^{3-2} = 4x^1 = 4x \qquad \frac{-2x^4}{10x^7} = \frac{-1x^{4-7}}{5} = \frac{-1x^{-3}}{5} = \frac{-1}{5x^3}$$

11. Raising an exponent to an exponent

Raise the coefficient to the indicated exponent. Multiply the exponents to get the variable's new exponent.

$$\left(ax^n\right)^m = a^m x^{nm}$$

Examples

$$\left(x^2\right)^3 = x^{2\cdot 3} = x^6 \qquad \left(4x^3\right)^3 = 4^3 x^{3\cdot 3} = 64x^9$$

12. Fractional (rational) exponents

The expression

$$x^{m/n}$$

indicates that the n^{th} root of x is to be taken and the result raised to the m^{th} power:

$$x^{m/n} = \left(x^{1/n}\right)^m = \left(\sqrt[n]{x}\right)^m$$

Examples

$$4^{1/2} = \sqrt{4} = 2 \qquad 8^{2/3} = \left(8^{1/3}\right)^2 = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$$

$$25^{-1/2} = \frac{1}{25^{1/2}} = \frac{1}{\sqrt{25}} = \frac{1}{5}$$

13. Like terms

Terms having the same variable and the same exponent are like terms.

Examples

$3x$ and $-4x$, $\quad 5n, \frac{n}{2}$ and $-n$, $\quad -7xy$ and $3xy$, $\quad z$ and $\frac{-1}{6}$

Notice that constants are like terms whose variable has an exponent of zero:

$4x^0 = 4 \cdot 1 = 4$

14. Adding like terms (monomials)

Add (combine) only like terms by adding their coefficients and leaving the bases and exponents as they were.

Example

Combine like terms:

$-5x^2 + 3xy + 2x + 15x^2 + 7xy - 3x = 10x^2 + 10xy - x$

15. Subtraction of polynomials

See Rules 2 and 14.

Example

Subtract $+12x^2 - 7x$ from $-3x^2 - 9x$.

$-3x^2 - 9x$

$\underline{-12x^2 + 7x}$ (Change each sign of the subtrahend and follow the rules for the addition of

$-15x^2 - 2x$ signed numbers, page 14.)

16. Multiplication of a monomial by a polynomial (removing parentheses)

Multiply the monomial by each term of the polynomial (distributive property), and apply Rule 9 (above) to each of the multiplications.

Example

$-3x^2y^3(5x^2y - 2xy^2 - 1)$

$= -3x^2y^3(5x^2y) - 3x^2y^3(-2xy^2) - 3x^2y^3(-1)$

$= -15x^4y^4 + 6x^3y^5 + 3x^2y^3$

17. Multiplication of a polynomial by a polynomial

Multiply each term of one polynomial by each term in the other polynomial and then collect like terms.

Example

$(3x - 2)(5x^2 + 7x - 1)$

$= 3x(5x^2 + 7x - 1) - 2(5x^2 + 7x - 1)$

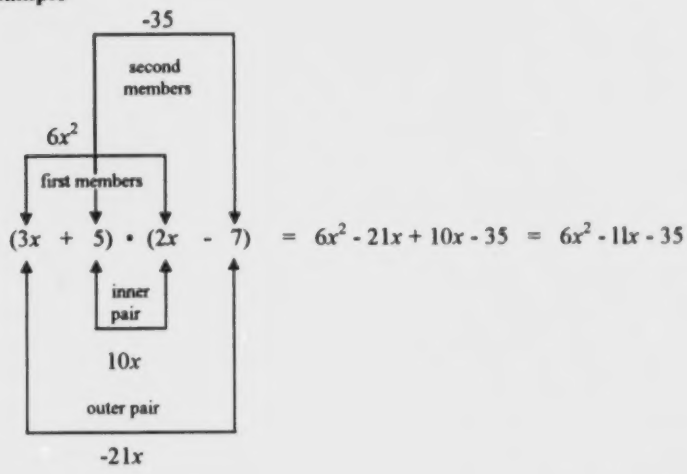
$= 15x^3 + 21x^2 - 3x - 10x^2 - 14x + 2$

$= 15x^3 + 11x^2 - 17x + 2$

18. Multiplication of two binomials

In the multiplication of two binomials, the procedure of Rule 17 can be summarized in the following four steps: (1) Multiply the pair of first members; (2) multiply the outer pair; (3) multiply the inner pair; (4) multiply the pair of second members. Then combine like terms (if any).

Example



$= 6x^2 - 21x + 10x - 35 = 6x^2 - 11x - 35$

Note: To remember which terms to multiply, remember the word FOIL (First, Outer, Inner, Last).

19. Factoring

There are three frequent factoring situations in algebraic expressions:

A. Common factor in a polynomial:
$18xy^3z - 6x^2y + 9xyz^2 = 3xy(6y^2z - 2x + 3z^2)$

B. Difference of two perfect squares $[a^2 - b^2 = (a + b)(a - b)]$:
$16x^6 - 9y^2 = (4x^3 - 3y)(4x^3 + 3y)$

C. Trinomial:
$6x^2 - x - 15 = (2x + 3)(3x - 5)$
(Factoring a trinomial of this type is a trial-and-error process in which you try various combinations of the factors of the first and last terms, testing each combination by rule 10. There may not be any rational factors.)

20. Evaluating a formula

Merely substitute the appropriate values for each of the variables.

Example

The formula for the volume of a cone is:

$$V = \frac{1}{3}\pi r^2 h$$

Find V, if $\pi = \frac{22}{7}$, $r = 7$, and $h = 12$.

7 4

$$V = \frac{1}{3} \bullet \frac{22}{7} \cdot (7)^2 \cdot (12)$$

$$V = (22) \bullet (7) \bullet (4) = 616$$

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21. Solving first-degree equations

Example 1

Solve for x:

$51 - 5x = 5x + 9 - 4x$

Combine like terms on each side of the equation: $51 - 5x = x + 9$

Add $+5x$ to both sides: $+5x = +5x$

$51 = 6x + 9$

Add -9 to both sides: $-9 = -9$

$42 = 6x$

Multiply both sides by $\frac{1}{6}$: $\frac{1}{6}(42) = \frac{1}{6}(6x)$

$7 = x$

Example 2

Solve for x:

$\frac{x}{3} + 7 = \frac{x}{5} - 3$

Multiply both sides of the equation by the lowest common denominator (15): $15\left(\frac{x}{3}\right) + 15(7) = 15\left(\frac{x}{5}\right) - 15(3)$

or:

$5x + 105 = 3x - 45$

Add $(-3x)$ to both sides: $-3x = -3x$

Add (-105) to each side:

$2x + 105 = -45$

$-105 = -105$

$2x = -150$

Multiply both sides by $\frac{1}{2}$: $\frac{1}{2}(2x) = \frac{1}{2}(-150)$

$x = -75$

Example 3

Solve for x:

$\frac{15}{4x} = \frac{1}{8}$

When there is only one fraction on either side of the equation, we may cross multiply:

$\frac{15}{4x} \times \frac{1}{8}$

$(4x)(1) = (15)(8)$

$x = 30$

Note: In this case, one may divide (cancel) by a common factor in any of the following directions:

$\frac{a}{b} = \frac{c}{d}$

$\frac{15}{4x} = \frac{1}{8}$

$x \quad 2$

$x = 30$

22. Solving a pair of linear equations simultaneously

Example

Solve for x and y:

$5x - 2y = 8$ (1)

$3x - y = 5$ (2)

Method 1: Substitution

Solve for y in Equation (2):

$y = 3x - 5$ (3)

Substitute this value for y in Equation (1):

$5x - 2(3x - 5) = 8$

Solve for x:

$5x - 6x + 10 = 8$

$-x + 10 = 8$

$-x = -2$

$x = 2$

Substitute this value of x in Equation (3) and solve for y:

$y = 3(2) - 5$

$y = 1$

Check: Substitute $x = 2$ and $y = 1$

in Equation (1)	in Equation (2)
$5(2) - 2(1) \stackrel{?}{=} 8$	$3(2) - 1 \stackrel{?}{=} 5$
$8 = 8$	$5 = 5$

Method 2: Addition

$5x - 2y = 8$ (1)

$3x - y = 5$ (2)

Multiply one (or both) equations by factors that will result in a pair of like terms with the same coefficient (absolute value) and opposite signs. An easy way to do this in this case is to multiply Equation (2) by -2:

$-2(3x - y) = -2(5)$

$-6x + 2y = -10$ (3)

Add Equations (1) and (3):

$$\begin{array}{r} 5x - 2y = 8 \\ \underline{-6x + 2y = -10} \\ -x \quad\quad = -2 \\ x = 2 \end{array}$$

Substitute this value of x in Equation (2):

$3(2) - y = 5$

$6 - y = 5$

$y = 1$

23. Solving a second-degree equation (quadratic)

In order to solve a (factorable) quadratic equation, collect all terms on one side of the equation and have zero on the other side, factor the nonzero side, and set each factor equal to zero. Then solve each linear equation to get two solutions for the variable. (Note: In order for a product $ab = 0$, either $a = 0$, or $b = 0$.)

Example

Solve for x:

$6x^2 + 9x = 6$

Set the terms equal to zero (i.e., add -6 to both sides): $6x^2 + 9x - 6 = 0$

Factor the trinomial: $(2x - 1)(3x + 6) = 0$

Set each factor equal to zero: $2x - 1 = 0 \quad \| \quad 3x + 6 = 0$

Solve each linear (first degree) equation: $2x = 1 \quad \| \quad 3x = -6$

$x = \frac{1}{2} \quad \| \quad x = -2$

24. Properties of inequalities

For real numbers a, b, m, n, and k, the following properties are true:

A. If $m > n$ and $a \geq b$, then $m + a > n + b$.

B. If $m > n$, then (1) $m - a > n - a$, and (2) $a - m < a - n$.

C. If $m > n$ and $a > 0$, then $am > an$.

D. If $m > n$ and $a > 0$, then (1) $\frac{m}{a} > \frac{n}{a}$, and (2) $\frac{a}{m} < \frac{a}{n}$.

E. If $n = m + k$ and $k > 0$, then $n > m$.

F. If a, b, m, and n are positive, and $\frac{a}{b} > \frac{m}{n}$, then $an > bm$.

25. Solving linear inequalities

Basically, a linear inequality is solved like a linear equation, with one major *exception*: When both sides of an inequality are multiplied or divided by a negative number, the order of the inequality changes.

Example

Solve for x:

$5 - 3x > 17$

Add -5 to both sides of the inequality to get:

$-3x > 12$

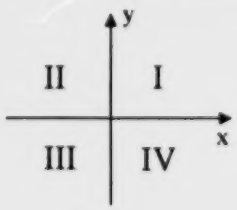
Multiply both sides of the inequality by $-\frac{1}{3}$ to get:

$x < -4$

Note: In the last step (which involved multiplication by a negative number) the *order* of the inequality changed.

26. x, y-coordinate system

Number lines intersect at right angles to form the x- and y-axes and the four quadrants of the x, y-coordinate system (also called the Cartesian plane). The quadrants are numbered as shown.



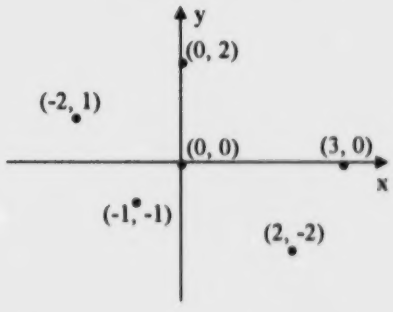
27. Ordered pair

A point on the x, y-plane is named as an ordered pair, with its x-coordinate first and its y-coordinate second.

$$(x, y)$$

The x- and y-axes intersect at the point (0, 0), the origin.

Examples



28. Linear equations

Linear equations contain the variables x and/or y; in a linear equation, the exponent of x and y is 1, and the x and y terms are added or subtracted. One way to represent a linear equation is:

$$Ax + By = C$$

in which A, B, and C are constants.

Examples

$3x + y = 1$ $\quad -x = 2$ $\quad 7y = 3$ $\quad y = -x + 4$

Examples of equations that are *not* linear:

$xy = 1$ $\quad x^2 + y^3 = 4$ $\quad \frac{1}{x} = 2y$

29. Graphing a linear equation by finding two points

As its name implies, a linear equation represents a line. Two points are needed to graph any line. To determine two points (x_1, y_1) and (x_2, y_2) from a given linear equation, proceed as in the following example.

Example

First solve the equation for y in terms of x:

$$4x + y = 2$$
$$y = -4x + 2$$

Then substitute any value in place of x and calculate y. For example, let $x = 0$, so

$$y = -4 \cdot (0) + 2 = 2$$

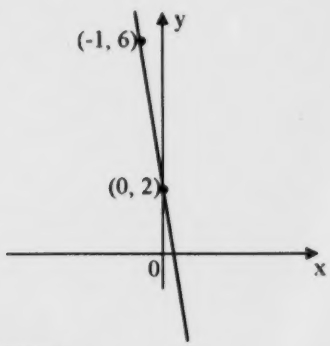
Thus $(x_1, y_1) = (0, 2)$.

Next, choose another value for x and calculate a second value for y.
For example, let $x = -1$, so

$$y = -4 \cdot (-1) + 2 = 6$$

Thus $(x_2, y_2) = (-1, 6)$.

Graph the points (0, 2) and (-1, 6) on the x, y-plane and draw the line through these points.



30. Midpoint of a line segment

Given the line segment whose endpoints are (x_1, y_1) and (x_2, y_2), the midpoint of the line segment is given by:

$$(x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Notice that x_m is the average of x_1 and x_2. Likewise, y_m is the average of y_1 and y_2.

Example

The midpoint of the line segment whose endpoints are $(6, -5)$ and $(-2, 0)$ is

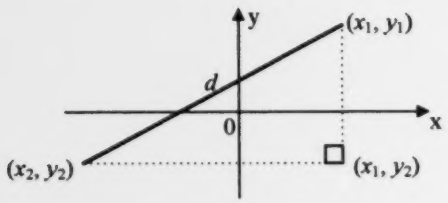
$$(x_m, y_m) = \left(\frac{6 + (-2)}{2}, \frac{-5 + 0}{2} \right) = \left(2, \frac{-5}{2} \right)$$

31. Distance between two points (length of a line segment)

The distance between two points (x_1, y_1) and (x_2, y_2) is given by:

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

This formula derives from the fact that a right triangle constructed from the two given points has leg lengths equal to $|x_1 - x_2|$ and $|y_1 - y_2|$, as shown:



Then, from the Pythagorean Theorem,

$$d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$$

in which d is the length of the hypotenuse. Solving for d gives the distance formula.

32. Slope of a line

For a line containing the points (x_1, y_1) and (x_2, y_2), the slope of the line is the ratio:

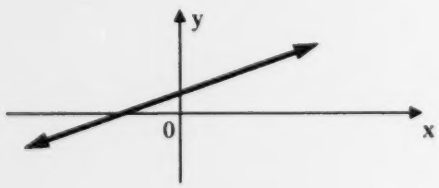
$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

Example

If $(x_1, y_1) = (0, 7)$ and $(x_2, y_2) = (-3, 5)$, then $m = \frac{7 - 5}{0 - (-3)} = \frac{2}{3}$.

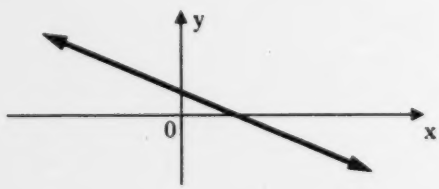
33. Positive slope

A line with a positive slope has the general appearance:



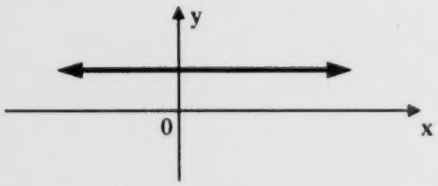
34. Negative slope

A line with a negative slope has the general appearance:



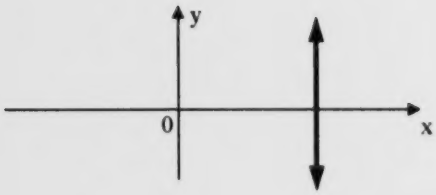
35. Zero slope

A horizontal line has a slope of zero:



36. No slope (or undefined slope)

A vertical line has no slope:



37. Slope-intercept form of a line

Solving a linear equation for y in terms of x gives the slope-intercept form of the line:

$$y = mx + b$$

in which m, the coefficient of x, is the *slope of the line*, and b, the value of y when $x = 0$, is the line's *y-intercept*:

$$y = m\cdot(0) + b = 0 + b = b$$

Examples

Solving $2x + 3y = 12$ for y gives:

$$y = \frac{-2}{3}\cdot x + 4$$

Slope $m = \frac{-2}{3}$ and y-intercept $b = 4$.

Solving $x + y = 6$ for y gives:

$$y = -x + 6$$

in which $m = -1$ and $b = 6$.

38. Graphing the slope-intercept form of a line

Plot the y-intercept $(0, b)$ first. Use the slope m to arrive at a second point.
Starting at $(0, b)$:

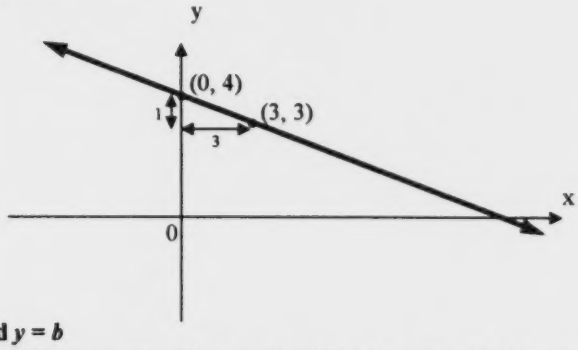
If m is positive, move *up* the number of spaces in m's numerator and *right* the number of spaces in m's denominator. Plot this point.

If m is negative, move *down* the number of spaces in m's numerator and *right* the number of spaces in m's denominator. Plot this point.

Draw the line through the two points plotted.

Example

$$y = \frac{-x}{3} + 4$$

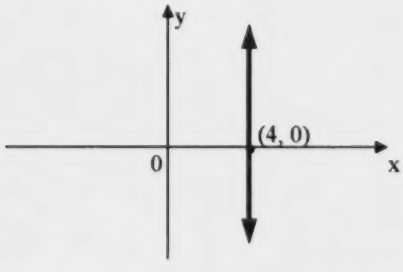


39. Graphing lines of the form $x = a$ and $y = b$

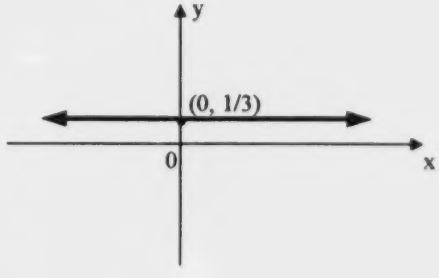
A linear equation having only one variable represents either a vertical line $(x = a)$ or a horizontal line $(y = b)$. Draw a vertical line through a point $(a, 0)$. Draw a horizontal line through a point $(0, b)$.

Examples

$x - 4 = 0$
$x = 4$



$3y = 1$
$y = 1/3$



A BRIEF REVIEW OF SOME ESSENTIALS OF PROBABILITY

1. Experiment and outcome

An experiment in probability theory generally refers to any observable activity in which one outcome is equally as likely to occur as another outcome.

Examples

Experiment: Tossing a fair coin
Outcome: Heads or tails

Experiment: Rolling a fair die
Outcome: The number of dots that turn up

Experiment: Choosing a playing card at random from a deck of cards
Outcome: The suit and value of the card

2. Sample space

The set of all possible outcomes in an experiment.

Examples

The experiment of tossing a coin has sample space {heads, tails}.

The experiment of rolling a die has sample space {1, 2, 3, 4, 5, 6}.

The experiment of choosing a playing card has the sample space in which all 52 cards are listed according to face value and suit: {2 of diamonds, 3 of diamonds, . . ., 2 of clubs, 3 of clubs, . . .}.

3. Event

The particular outcome that the person performing the experiment seeks.

Examples

The event of a tail turning up when a fair coin is tossed.

The event of an even number turning up when a fair die is rolled.

The event of a queen being chosen randomly from a deck of cards.

4. Probability of an event

Given a sample space having n equally likely outcomes and s ways for a particular event to occur, the probability of that event occurring is:

$$p = \frac{s}{n}$$

For any event, since $0 \le s \le n$, $0 \le p \le 1$.

Examples

The probability of a tail turning up when a fair coin is tossed is ½, or 0.5.

The probability of an even number turning up when a fair die is rolled is 3/6, or 0.5.

The probability of either an even or an odd number turning up when a fair die is rolled is 6/6, or 1. This is a *certain* event.

The probability of a 7 turning up when a fair die is rolled is 0. This is an *impossible* event.

The probability of a queen being randomly chosen from a deck of cards is 4/52, or 1/13.

The probability of a girl being chosen at random from a class of 8 girls and 11 boys is 8/19. (The sample space has 19 equally likely outcomes since there are 19 children.)

5. Counting (multiplication principle)

Suppose there are n_1 *different* ways for an event to occur. Suppose also that there are n_2 ways for a second event to occur. Then the number of ways for the sequence of the first event followed by the second to occur is given by:

$$n_1 \cdot n_2$$

In general, the number of ways for a sequence of events to occur is given by:

$$n_1 \cdot n_2 \cdot \ldots \cdot n_m$$

in which n_1, n_2,... are the number of ways for successive events to occur and m is the number of events in the sequence.

Examples

The number of ways to pick 5 cards from a deck of playing cards (without replacing a card once it is chosen) is:

$$52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 = 311{,}875{,}200$$

The number of ways to arrange the letters in the name Alfred is:

$$6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$$

(Notice that all the letters are different in this name.)

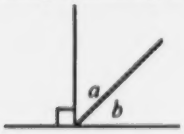
The number of ways for 10 people to line up is:

$$10! = 10 \cdot 9 \cdot 8 \cdot 7 \cdot \ldots \cdot 2 \cdot 1 = 3{,}628{,}800$$

Definition of $n!$ $\qquad n! = n \bullet (n-1) \bullet (n-2) \bullet \cdots \bullet 2 \bullet 1$

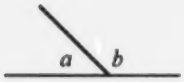
A BRIEF REVIEW OF SOME ESSENTIALS OF GEOMETRY

1. Complementary angles (two angles whose measures have a sum of 90^0)



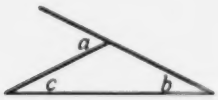
$m\angle a + m\angle b = 90^0$

2. Supplementary angles (two angles whose measures have a sum of 180^0)



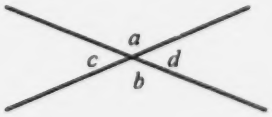
$m\angle a + m\angle b = 180^0$

3. An exterior angle of a triangle equals in measure the sum of the measures of the two remote interior angles.



$m\angle a = m\angle b + m\angle c$

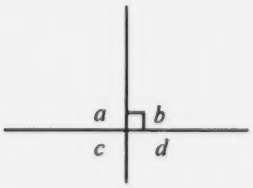
4. Vertical angles have equal measure.



$m\angle a = m\angle b$

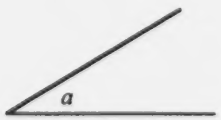
$m\angle c = m\angle d$

5. Perpendicular lines intersect to form right angles.



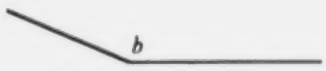
$m\angle a = m\angle b = m\angle c = m\angle d = 90^0$

6. An acute angle has measure greater than 0^0 and less than 90^0.



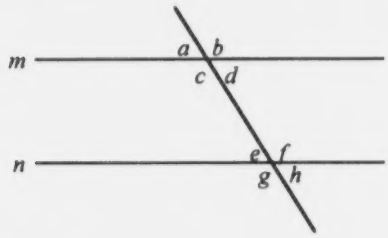
$\angle a$ is acute

7. An obtuse angle has measure greater than 90^0 and less than 180^0.



$\angle b$ is obtuse

8. Parallel lines (m and n)



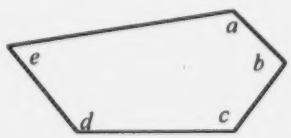
Alternate interior angles have equal measure.
$m\angle c = m\angle f$ and $m\angle d = m\angle e$

Corresponding angles have equal measure.
$m\angle a = m\angle e$, $m\angle b = m\angle f$, $m\angle c = m\angle g$, $m\angle d = m\angle h$

Consecutive interior angles are supplementary.
$m\angle c + m\angle e = 180^0$, $m\angle d + m\angle f = 180^0$

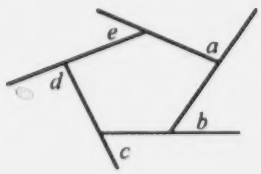
9. The sum of the measures of the interior angles of any polygon of n sides is $(n - 2)\ 180^0$.
(For example: The sum of the measures of the angles of a triangle is $(3 - 2)\ 180^0 = 180^0$.)

Example



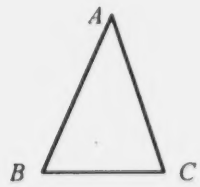
$n = 5$, therefore:
$m\angle a + m\angle b + m\angle c + m\angle d + m\angle e = (5 - 2)\ 180^0 = 540^0$

10. The sum of the measures of the exterior angles of any polygon is 360^0.



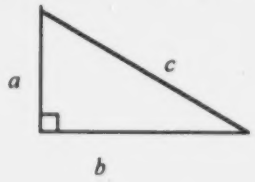
$m\angle a + m\angle b + m\angle c + m\angle d + m\angle e = 360^0$

11. The base angles of an isosceles triangle have equal measure (the converse is also true).



If $AB = AC$, then $m\angle B = m\angle C$.
If $m\angle B = m\angle C$, then $AB = AC$.

12. The Pythagorean Theorem:
The sum of the squares of the lengths of the legs of a right triangle equals the square of the length of the hypotenuse.



$a^2 + b^2 = c^2$

Given any two sides of a right triangle, we can always find the third side by using the Pythagorean theorem. However, problems involving right triangles (especially those on the SAT I) almost always use triangles with whole numbers (or whole number ratios) as sides. Therefore, you can save valuable time if you have memorized the more frequently used sets of integers that satisfy the Pythagorean relationship. Then when you spot two numbers from one of these sets, you know the third without doing any arithmetic. The following is a list of the Pythagorean "triples" you should learn to recognize on sight. Remember, too, that multiples of these sets also satisfy the relation.

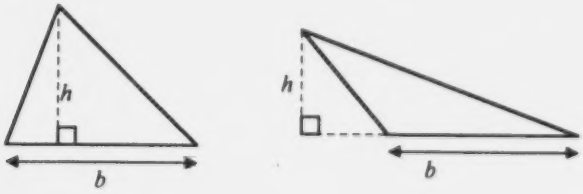
3-4-5 (6-8-10; $\frac{3}{5}$-$\frac{4}{5}$-$\frac{5}{5}$; 9π-12π-15π; etc.)

5-12-13	9-40-41
8-15-17	12-35-37
7-24-25	11-60-61
20-21-29	

Example
In solving a problem, you have a right triangle in which one leg has length 10 and the hypotenuse has length 26. Therefore, the other leg has length 24, since the triple 10-24-26 is a multiple of (twice) the Pythagorean triple 5-12-13.

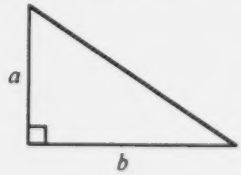
Note
When reduced to their lowest terms, no two members of a Pythagorean triple have a common factor. Therefore, if the two known sides of the triangle have a common factor (as in the example of 10 and 26), reduce by that factor to help identify the Pythagorean triple (if any) to which the sides belong. (Don't forget to multiply the length of the third side in the reduced triple by the same factor.)

13. The area of a triangle is equal to one half the product of the lengths of a base and the altitude drawn to that base.



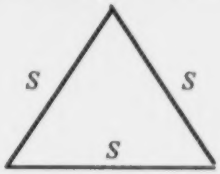
$\text{area} = \frac{1}{2}\,bh$

14. The area of a right triangle is equal to one half the product of the lengths of its legs.



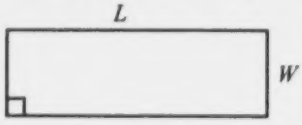
$\text{area} = \frac{1}{2}\,ab$

15. The area of an equilateral triangle with side length S is equal to $\frac{S^2\sqrt{3}}{4}$.



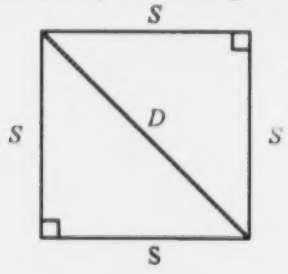
$$\text{area} = \frac{S^2\sqrt{3}}{4}$$

16. The area of a rectangle is equal to the product of its length and width.



$$\text{area} = LW$$

17. The area of a square is equal to the length of a side squared, OR one-half the length of the diagonal squared.



$$\text{area} = S^2$$

OR

$$\text{area} = \frac{1}{2}D^2$$

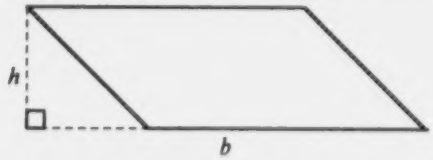
Also:

$$D = S\sqrt{2}$$

and:

$$S = \frac{1}{2}D\sqrt{2}$$

18. The area of a parallelogram is equal to the product of the lengths of its base and altitude drawn to that base.



$$\text{area} = bh$$

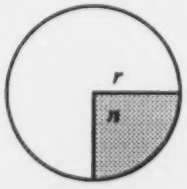
19. The circumference of a circle with radius r is $2\pi r$.

$$C = 2\pi r$$

20. The area of a circle with radius r is $\pi \cdot r^2$.

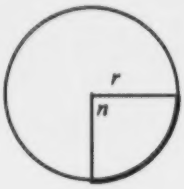
$$\text{area} = \pi r^2$$

21. Since the **area of a sector** of a circle, radius r, is $\frac{n^0}{360^0}$ of the area of the circle (n is the measure of the central angle of the sector), we get:



$$\text{area of sector} = \frac{n^0}{360^0} \cdot \pi r^2$$

22. Since the **length of an arc** of a circle, radius r, is $\frac{n^0}{360^0}$ of the circumference of the circle (n is the measure of the arc), we get:



$$\text{length of arc} = \frac{n^0}{360^0} \cdot 2\pi r$$

Note:
From rules 20 and 21 we get the following proportion:

$$\frac{n^0}{360^0} = \frac{\text{area of sector}}{\text{area of circle}} = \frac{\text{length of arc}}{\text{circumference}}$$

23. For **any similar polygons** I and II, we may set up the following proportions:

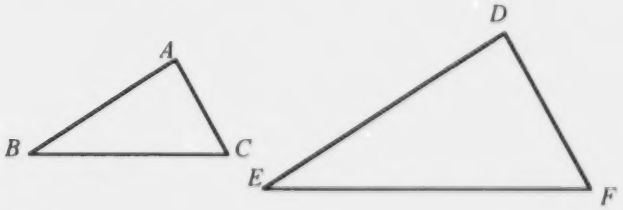
$$\frac{\text{Area I}}{\text{Area II}} = \frac{(\text{Side I})^2}{(\text{Corresponding side of II})^2} = \frac{(\text{Any linear part I})^2}{(\text{Corresponding linear part II})^2}$$

$$\frac{\text{area I}}{\text{area II}} = \frac{a^2}{A^2}$$

Similarly for **3-dimensional similar figures**:
Two corresponding volumes are to each other as the cubes of their corresponding edges. That is,

$$\frac{\text{Volume I}}{\text{Volume II}} = \frac{(\text{Edge I})^3}{(\text{Corresponding edge II})^3}$$

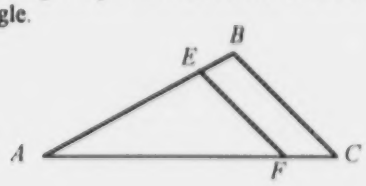
24. **Two triangles are similar** if (1) the measures of their corresponding angles are equal and (2) the lengths of their corresponding sides are in proportion. If (1) is true, then (2) is also true, and conversely, if (2) is true, then (1) is also true.



ΔABC is similar to ΔDEF
$m\angle A = m\angle D,\ m\angle B = m\angle E,\ m\angle C = m\angle F$

$$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$$

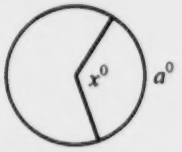
Note:
If a line in a triangle is parallel to one side of the triangle, then it forms a triangle similar to the original triangle.



If $\overline{EF}$ is parallel to $\overline{BC}$, ΔABC is similar to ΔAEF.

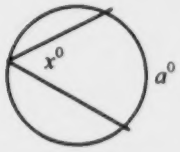
25. Circles and angle measurement

A. The measure of a central angle is equal to the measure of its intercepted arc.

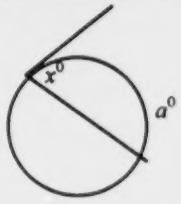


$x = a$

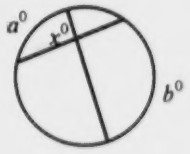
B. The measure of an angle with vertex on the circle (i.e., formed by either two chords intersecting on the circle, or a tangent and a chord intersecting on the circle) is equal to one half the measure of its intercepted arc.



$x = \frac{1}{2}a$

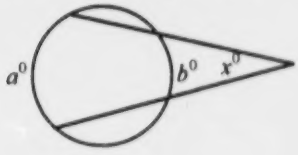


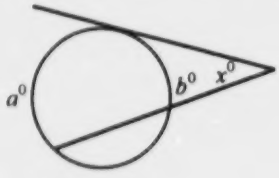
C. The measure of an angle formed by two chords intersecting inside the circle equals one half the sum of the measures of the two intercepted arcs.



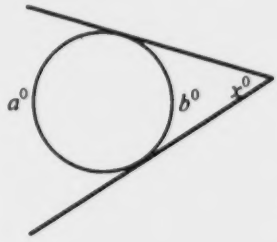
$x = \frac{1}{2}(a + b)$

D. The measure of an angle whose vertex is outside the circle is equal to one-half the difference of the measures of the two intercepted arcs .



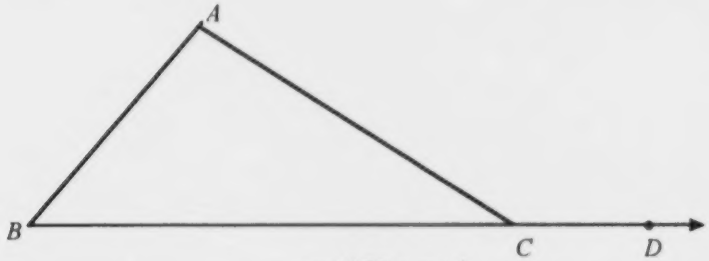


$x = \frac{1}{2}(a - b)$



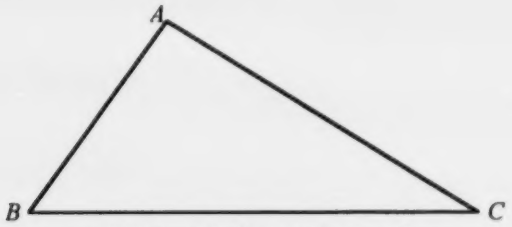
26. Geometric inequalities

A. The measure of an exterior angle of a triangle is greater than the measure of either remote interior angle.



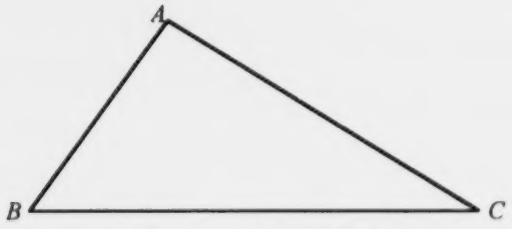
$m\angle ACD > m\angle A$
$m\angle ACD > m\angle B$

B. If two sides of a triangle are not congruent, then the angles opposite those sides are not congruent, the angle with greater measure being opposite the longer side.



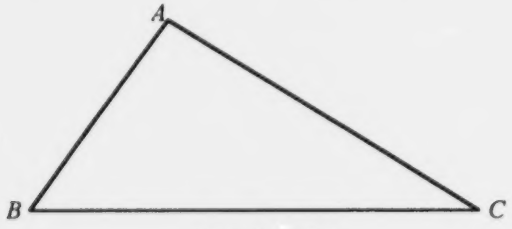
If $AC > AB$, then $m\angle B > m\angle C$.

C. If two angles of a triangle are not congruent, then the sides opposite those angles are not congruent, the longer side being opposite the angle with the greater measure.



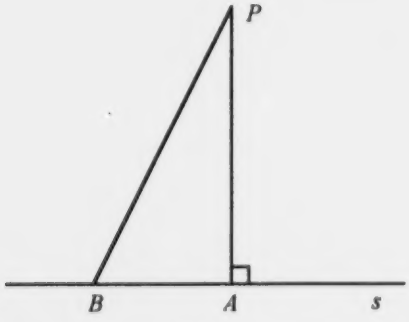
If $m\angle A > m\angle C$, then $BC > AB$.

D. The sum of the lengths of any two sides of a triangle is greater than the length of the third side.



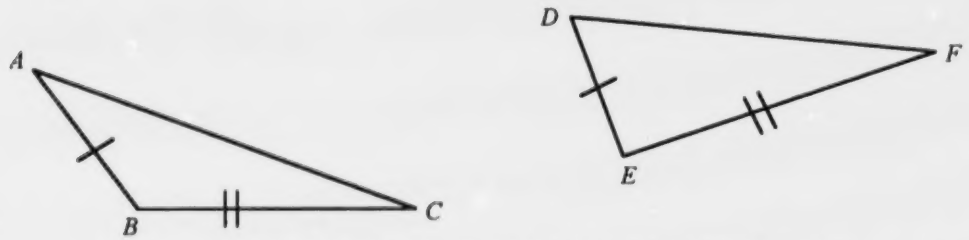
$AB + AC > BC$
$AB + BC > AC$
$AC + BC > AB$

E. The shortest distance between a given point and a given line is the length of the perpendicular segment from the point to the line.



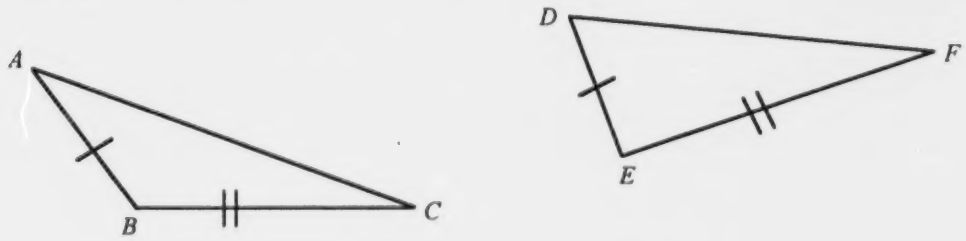
If $\overline{PA}$ is perpendicular to line s, and B is any point on line s other than A, then $PB > PA$.

F. If two sides of a triangle are congruent respectively to two sides of a second triangle, and the measure of the included angle of the first triangle is greater than the measure of the included angle of the second triangle, then the measure of the third side of the first triangle is greater than the measure of the third side of the second triangle.



If $AB = DE$ and $BC = EF$, and if $m\angle B > m\angle E$, then $AC > DF$.

G. If two sides of one triangle are congruent respectively to two sides of a second triangle, and the measure of the third side of the first triangle is greater than the measure of the third side of the second triangle, then the measure of the included angle of the first triangle is greater than the measure of the included angle of the second triangle.



If $AB = DE$ and $BC = EF$, and if $AC > DF$, then $m\angle B > m\angle E$.

II. Sample Multiple-Choice Problems

As we have stated before, the purpose of the Mathematics Section of the SAT I is *not* to find out how much math you know or how good you are at solving routine problems but to test your understanding of fundamental mathematical ideas and your ability to apply those ideas quickly to novel mathematical situations. Speed in doing mathematical operations is important, but speed in finding the best way to attack the problem is even more essential. If you find a good approach, the necessary arithmetic or algebra will generally be quite simple. If, on the other hand, you take an unimaginative approach, you are likely to get bogged down in time-consuming calculations that rob you of vital minutes and open the way to mechanical errors.

When taking the SAT I, remain aware of the time factor. If after a reasonable amount of thought you do not see a likely method of solving a problem, skip it; come back to it later if you have time. Likewise, if you find yourself involved in a long computation or struggling with several unknowns in a series of equations, the chances are that you are on the wrong track; give it up for the time being and go on to the next question.

The following sample problems illustrate the kinds of shortcut ideas you should be looking for. Try to solve each problem first on your own, without reading the explanation. Then compare the given solution with the method you used. In connection with this matter of method, remember that most problems can be solved in two or more different ways. If you find a method of your own that is not the same as the one we suggest, that is all right so long as your method works. In such cases, decide which method seems easier for you, and remember it for the next problem of that kind you encounter.

SAMPLE PROBLEM 1

The approximate volume in cubic feet of a high round-top haystack may be determined by the following formula:

$$V = (.52M - .44W)WL$$

In this formula, W and L represent the stack's width and length in feet, and M is the "over" measurement obtained by throwing a rope over the stack from a point on the ground on one side of the stack to the corresponding point on the ground on the opposite side. A stack of alfalfa, which is 4 months old, has an average width of 20 feet and is 40 feet long. Its "over" measurement is 40 feet. What is the approximate number of tons of alfalfa in the stack if alfalfa that has settled for more than 90 days runs around 480 cubic feet per ton?

(A) 2 (B) 8 (C) 20 (D) 30 (E) 40

Analysis

This problem illustrates three points:

1. *The importance of reading a problem carefully to determine exactly what information is given and what is required.* Did you realize that a stack that is "4 months old" is one that has "settled for more than 90 days" and that the detailed definition of M, the "over" measurement, does not have to be used at all? This problem turns out to be a simple case of substituting numbers in a formula.

2. *The mechanics of substituting values in a formula and working out the result.* You discover by reading the problem that M = 40, W = 20, and L = 40. (Check to make sure that these quantities are expressed in the proper units. If the measurements were in yards, for example, you would have to convert to feet before using the formula.)

$$V = (.52 \times 40 - .44 \times 20) \times 20 \times 40$$

The part in parentheses must be worked out first (see "Order of Operations," page 9):

$$.52 \times 40 - .44 \times 20 = 20.8 - 8.8 = 12$$

Therefore, $V = 12 \times 20 \times 40$

3. *The need to avoid unnecessary calculations.* The question does not call for the volume. It asks for the number of tons, and it tells you that the volume must be divided by 480 to find the weight in tons.
Therefore:

$$\text{Tons} = \frac{V}{480} = \frac{12 \times 20 \times 40}{480} = 20$$

Notice how easy the arithmetic is and how conveniently the numbers work out. This is characteristic of SAT I problems. You may, however, also use a calculator to get the answer.

Having worked the problem and obtained an answer, you look for your answer among the given choices. In this case, it is Answer (C), and you mark your answer sheet accordingly.

There is an interesting and important point worth noting about multiple-choice math questions. If you do a problem incorrectly, you will often be alerted to that fact by not finding your answer among the given choices. This gives you a second chance to do it right. On the other hand, finding your answer among those listed is no guarantee that you have done the problem correctly; you may have made a common type of error that the examiners allowed for when making up the wrong choices. Carelessness in your arithmetic could lead to Answer (A), 2, instead of 20; incorrect substitution of values could lead to one or another of the other wrong answers.

So even when you do find your answer among the choices listed, take a few seconds to run through your computations as a final check. When you do so, vary your procedure somewhat. Multiply factors in a different sequence, or add down instead of up to avoid repetition of an error.

In some types of problems, you can check your answer for reasonableness. For example, if you are asked for the area of the shaded region of a given figure, you know it has to be less than the area of the whole figure, and you can usually judge whether it is more or less than half that area. If your answer doesn't agree with your estimate of what it should be, or if it seems unreasonable in some other way, review your method of solution and your computations to look for an error.

Occasionally, one of the answer choices is "none of these." Now if you don't find your result among the answers, you may be right or wrong. Again, it is advisable to check your method and calculations (even if you used a calculator) before marking your answer sheet.

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SAMPLE PROBLEM 2

Evaluate:

$$\frac{24690}{(12346)^2 - (12345)(12347)}$$

(A) 24690 (B) 12345 (C) 1 (D) 0 (E) none of these

Analysis

If you are a calculating wizard (or you are extremely adept at using a calculator), you can do the indicated multiplications, evaluate the denominator of the fraction, and get the answer. But if you are an ordinary mortal, this is going to take much more time than it is worth. It is much better to note that the numbers in the denominator form a *consecutive* series, so that if 12346 is represented by n, 12345 is n - 1 and 12347 is n + 1. You don't have to remember much algebra to suspect that something useful will come from using these expressions in place of the actual numbers. The denominator can be expressed as:

$n^2 - (n - 1)(n + 1)$

This readily becomes:

$n^2-(n^2 - 1) = n^2 - n^2 + 1 = 1$

In other words, the value of the denominator is 1, and the value of the fraction is the same as the numerator, 24690.

The answer is (A).

SAMPLE PROBLEM 3

Evaluate:

$3.1416 \times 7.5944 - 3.1416 \times 5.5944$

(A) 5.3692 (B) 6.5132 (C) 6.2832 (D) 6.1632 (E) approximately 120

Analysis

Here, too, you know better than to start multiplying these five-digit numbers and following the operations in order from left to right. Whenever you see a sum of products in an SAT I problem, look for a way to simplify by factoring. Sometimes you have to rearrange the terms first. In this case, the common factor 3.1416 is obvious. And as you may be able to see even before you do it, the remaining term is going to reduce to a small whole number:

$3.1416 (7.5944 - 5.5944) = 3.1416 (2) = 6.2832$
If you use a calculator, then watch out for the correct order of operations (see page 8).

The answer is (C).

SAMPLE PROBLEM 4

Which of the following is equivalent to the continued fraction: $\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{7}}}$**?**

(A) $\frac{21}{2}$ **(B)** $\frac{7}{22}$ **(C)** $\frac{15}{22}$ **(D)** $\frac{22}{15}$ **(E)** $\frac{2}{21}$

Analysis

Method 1

To simplify this continued fraction, we begin at the "end," by writing $2+\frac{1}{7}$ as an improper fraction: $\frac{15}{7}$,

so that the continued fraction now reads : $\cfrac{1}{1+\cfrac{1}{\frac{15}{7}}}$.

Since $\cfrac{1}{\frac{15}{7}} = 1\div\frac{15}{7} = 1 \bullet \frac{7}{15} = \frac{7}{15}$, the continued fraction is now equal to: $\cfrac{1}{1+\frac{7}{15}}$.

Repeating this process: $1 + \frac{7}{15} = \frac{15}{15} + \frac{7}{15} = \frac{22}{15}$.

Therefore: $\cfrac{1}{1+\frac{7}{15}} = \cfrac{1}{\frac{22}{15}} = 1 + \frac{22}{15} = 1 \bullet \frac{15}{22} = \frac{15}{22}$. Choice (C).

Method 2

Using a calculator, working from the end of continued fraction: $\frac{1}{7} = 1 \div 7 = .142857142$

$2 + \frac{1}{7} = 2 + .142857142 = 2.142857142$

$\cfrac{1}{2+\frac{1}{7}} = \cfrac{1}{2.142857142} = 1 + 2.142857142$ (or use "$\frac{1}{x}$" function on calculator)

$= .466666666$

$1 + \cfrac{1}{2+\frac{1}{7}} = 1 + .466666666 = 1.466666666$

$\cfrac{1}{1+\cfrac{1}{2+\frac{1}{7}}} = \cfrac{1}{1.466666666} = 1 + 1.466666666 = .681818182$ (or use "$\frac{1}{x}$" function on calculator)

This decimal result does not "look like" any of the choices given in the problem. The number .681818182 is actually a **repeating decimal** (.681818181. . .) rounded off to 9 places. We may write this decimal as $.6\overline{81}$. The bar over 81 indicates that the 81 repeats infinitely.

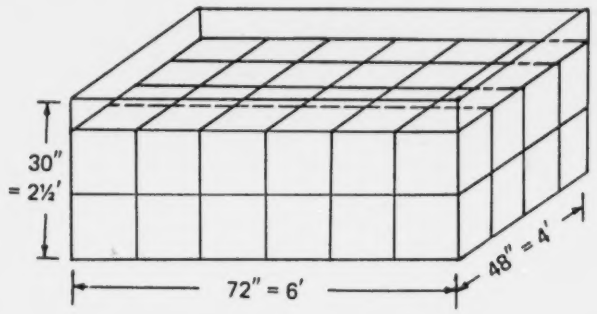
To convert this repeating decimal to a fraction we set it equal to x:

$$x = .6\overline{81}$$
$$10x = 6.8\overline{181}$$
$$1000x = 681.\overline{81}$$
$$1000x - 10x = 675$$
$$990x = 675$$
$$x = \frac{675}{990} = \frac{45 \bullet 15}{45 \bullet 22} = \frac{15}{22}.$$ Choice (C).

SAMPLE PROBLEM 5

What is the greatest number of cubes with 1-foot edges that may be packed into a box with inside dimensions of 30 inches X 48 inches X 72 inches?

(A) 30 (B) 48 (C) 60 (D) 72 (E) 120



Analysis

Here is a type of problem that calls for a little mathematical "common sense" in addition to some straightforward reasoning. Obviously, the first step should be to convert the dimensions of the box to feet, giving us dimensions of $2\frac{1}{2}$ feet X 4 feet X 6 feet. Since each cube has a volume of 1 cubic foot, it may seem that all we have to do is find the volume of the box in cubic feet. This volume is 60, but surely you can see that 60 1-foot cubes can't be put into this box. We can make two layers, each 4 X 6, but then there is a volume left over only $\frac{1}{2}$ foot deep, into which no cubes will fit unless we cut them up and the problem doesn't allow this. The answer is (B).

SAMPLE PROBLEM 6

What is the greatest number of cubes with 6-inch edges that may be packed into a box of dimensions 2 feet, 3 feet, and 5 feet?

(A) 30 (B) 15 (C) 60 (D) 240 (E) 300

Analysis

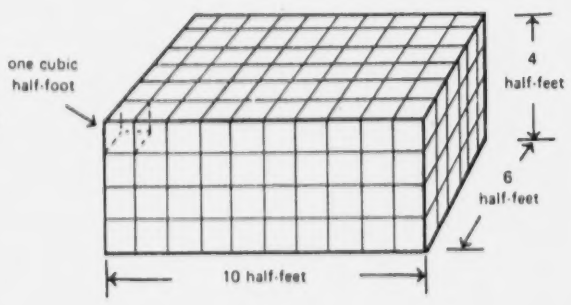
This is a variation on Sample Problem 5, in which we see that the box will be fully packed. The first question is What shall we do about the mixture of inches and feet? We will get involved with large numbers if we convert the dimensions to inches, so it is better to consider the cubes as having $\frac{1}{2}$-foot edges. Now, each cube has a volume of $\left(\frac{1}{2}\right)^3$ or $\frac{1}{8}$ cubic foot; we could calculate the volume of the box in cubic feet and divide by $\frac{1}{8}$ (multiply by 8) to find the number of cubes it will hold. The arithmetic is much easier, however, if we convert the dimensions of the box to "half-feet." The dimensions of the box are
now 4 half-feet, 6 half-feet, and 10 half-feet.
The volume in cubic half-feet is:

4 X 6 X 10 = 240

Since the volume of each cube is:

1 half-foot X 1 half-foot X 1 half-foot = 1 cubic half-foot,
the box will hold 240 such cubes.

The answer is (D).



SAMPLE PROBLEM 7

If a apples sell for d dollars, find the price in cents of b apples selling at the same rate.

(A) $\frac{100d}{b}$ **(B)** $\frac{100b}{d}$ **(C)** $\frac{100ab}{d}$ **(D)** $\frac{100ad}{b}$ **(E)** $\frac{100bd}{a}$

Analysis

Method 1

Problems of this kind may be handled by setting up a proportion. The difficulty lies in arranging the terms in the right order. The important requirement is to put corresponding terms in corresponding positions. But before we begin, let us note that the question asks for the result in cents; therefore, we must change the given price of d dollars to 100d cents.

One way to set up the proportion is as follows:

Let x = the price of b apples in cents. Then: $\frac{a}{b} = \frac{100d}{x}$.

Observe that a is in the numerator of one ratio, and its price is in the numerator of the other. Also, apples are on the left, cents on the right.
Now solve for x.

Cross multiply: $ax = 100bd$

Divide by a: $x = \frac{100bd}{a}$

The answer is (E).

The same result is obtained by putting the apples and their cost in each ratio, making sure they are in the same relative position:

$$\frac{a}{100d} = \frac{b}{x} \qquad ax = 100bd \qquad x = \frac{100bd}{a}$$

Method 2

Some students may be more comfortable if they find the expression for the cost of one apple, and then multiply by b to find the cost of b apples. The question then is What is the cost of one apple, if a apples cost 100d cents? That is, do you divide a by 100d, or vice versa?

The sure way to decide is to realize that the cost of an apple is "cents per apple." This tells us that we must divide cents by apples. The cost of one apple, then, is:

$$\frac{100d \text{ cents}}{a \text{ apples}} = \frac{100d}{a} \text{ cents per apple}$$

The cost of b apples is b times as much:

$$\text{cost} = b \cdot \frac{100d}{a} = \frac{100bd}{a}$$

There is not much to choose between these two approaches. The first is mechanical; it avoids "reasoning." The second is a "reasoned" method. Time is about the same for both. Use the one that you are most at ease with.

SAMPLE PROBLEM 8

What is Alice's hourly rate if she is paid $15.00 for 1 hour and 20 minutes of work?

(A) $10.00 (B) $11.25 (C) $12.00 (D) $12.25 (E) $13.00

Analysis

The first thing to do, in any event, is to change 1 hour and 20 minutes to $1\frac{1}{3}$ hours, or $\frac{4}{3}$ hours. Now there are several approaches, depending on whether you like algebra, simple proportions, or plain logic.

Method 1

Let x = Alice's hourly rate. Then:

$$\frac{4}{3}x = \$15.00$$
$$4x = \$45.00$$
$$x = \$11.25$$

The answer is (B).

Method 2

Again let x = Alice's hourly rate, but instead of writing an equation, set up a proportion, as in Sample Problem 7:

$$\frac{15 \text{ (dollars)}}{\frac{4}{3} \text{ (hours)}} = \frac{x \text{ (dollars)}}{1 \text{ (hours)}} \qquad \frac{4}{3}x = 15 \qquad x = 11.25$$

Method 3

Avoid algebra altogether, and reason as follows: If Alice is paid $15.00 for $\frac{4}{3}$ of an hour (*four* thirds of an hour), she gets one fourth as much for *one* third of an hour:

$$\frac{1}{4} \text{ of } \$15.00 = \$3.75$$

For *three* thirds of an hour (one hour) she will be paid

$3 \bullet \$3.75 = \$11.25$

SAMPLE PROBLEM 9

Arrange the fractions $\frac{2}{3}$, $\frac{5}{7}$, $\frac{7}{15}$, and $\frac{8}{11}$ in order from smallest to largest.

(A) $\frac{2}{3}, \frac{5}{7}, \frac{7}{15}, \frac{8}{11}$

(B) $\frac{2}{3}, \frac{5}{7}, \frac{8}{11}, \frac{7}{15}$

(C) $\frac{7}{15}, \frac{8}{11}, \frac{5}{7}, \frac{2}{3}$

(D) $\frac{7}{15}, \frac{2}{3}, \frac{5}{7}, \frac{8}{11}$

(E) none of these

Analysis

There are various ways of comparing fractions to determine their relative magnitudes. Three methods are described here, and you will have to decide for yourself which seems easiest. But whatever the method you are planning to use, the first thing to do with a problem of this kind is to see if there is any obvious way of sorting the fractions by size. In this case, you should notice that $\frac{7}{15}$ is less than $\frac{1}{2}$ whereas the other three fractions are greater than $\frac{1}{2}$. This immediately puts $\frac{7}{15}$ first in the desired sequence and eliminates it from further consideration. All we need to do is find the order of the other three.

Method 1

Fractions can be compared by inspection if they have the same denominator. Therefore, one approach is to convert the given fractions to equivalent fractions with a common denominator. We can do this for all three fractions at once, or we can compare them two at a time. In this problem, the three denominators are 3, 7, and 11, and their least common multiple is $3 \bullet 7 \bullet 11$, or 231. If your arithmetic is shaky, it will be better to compare the fractions in pairs than to convert them to this denominator. Let us begin by comparing $\frac{2}{3}$ and $\frac{5}{7}$:

$\frac{2}{3} = \frac{14}{21}$ $\quad$ $\frac{5}{7} = \frac{15}{21}$ $\quad$ clearly $\frac{5}{7}$ is larger than $\frac{2}{3}$ (larger by $\frac{1}{21}$).

Now compare $\frac{5}{7}$ with $\frac{8}{11}$:

$\frac{5}{7} = \frac{55}{77}$ $\quad$ $\frac{8}{11} = \frac{56}{77}$ $\quad$ clearly $\frac{8}{11}$ is larger than $\frac{5}{7}$.

We were lucky; if $\frac{8}{11}$ had turned out to be smaller than $\frac{5}{7}$, an additional comparison with $\frac{2}{3}$ would have been necessary.

As it is, since $\frac{8}{11}$ is larger than $\frac{5}{7}$, and $\frac{5}{7}$ is larger than $\frac{2}{3}$ (and all of them are larger than $\frac{7}{15}$), the required order is: $\frac{7}{15}, \frac{2}{3}, \frac{5}{7}, \frac{8}{11}$.

The answer is (D).

Method 2

Another method, which may be easier for those adept with a calculator, is to convert all the fractions to decimals by dividing the numerator by the denominator. Again, we can ignore $\frac{7}{15}$.

$\frac{2}{3} = .666666666$

$\frac{5}{7} = .714285714$

$\frac{8}{11} = .727272727$

The correct order is now obvious.

Method 3

If we have two fractions, $\frac{a}{b}$ **and** $\frac{c}{d}$, we can convert them to a common denominator as follows:

$\frac{a}{b} = \frac{ad}{bd}$ $\quad$ $\frac{c}{d} = \frac{bc}{bd}$

From this we can see that if a, b, c, and d are positive, $\frac{a}{b} > \frac{c}{d}$ **if and only if ad > bc.**. This gives us a direct way of comparing two positive fractions by simply cross-multiplying numerators and denominators and comparing the results. Let us apply this method to $\frac{2}{3}$ and $\frac{5}{7}$:

Since 5 • 3 is greater than 2 • 7, we see that the second fraction, $\frac{5}{7}$, must be larger than the first, $\frac{2}{3}$.

In similar fashion we can compare $\frac{5}{7}$ and $\frac{8}{11}$:

5 • 11 = 55 $\quad$ 8 • 7 = 56

Since 8 • 7 is greater than 5 • 11, $\frac{8}{11}$ is larger than $\frac{5}{7}$.

You can see that this method is really the same as Method 1, except that we don't bother to write the common denominator, but simply calculate the numerators.

Practice in Comparing Fractions

Since comparing fractions is chiefly a matter of speed and accuracy, you may wish to acquire some practice in doing it. The following exercises are offered for this purpose. Answers are given at the end of the exercise.

A. In each example, which fraction is larger?

1. $\frac{5}{6}, \frac{7}{8}$
2. $\frac{1}{3}, \frac{5}{14}$
3. $\frac{8}{11}, \frac{17}{23}$
4. $\frac{7}{11}, \frac{5}{8}$
5. $\frac{7}{9}, \frac{4}{5}$
6. $\frac{7}{12}, \frac{18}{31}$
7. $\frac{13}{17}, \frac{19}{25}$
8. $\frac{11}{19}, \frac{5}{9}$

B. Arrange the following sets of fractions in order of size, beginning with the largest:

9. $\frac{3}{4}, \frac{9}{13}, \frac{5}{7}$
10. $\frac{2}{5}, \frac{19}{48}, \frac{4}{11}$
11. $\frac{4}{9}, \frac{13}{28}, \frac{7}{15}$
12. $\frac{9}{11}, \frac{14}{17}, \frac{17}{21}, \frac{23}{28}$

Answers

1. $\frac{7}{8}$
2. $\frac{5}{14}$
3. $\frac{17}{23}$
4. $\frac{7}{11}$
5. $\frac{4}{5}$
6. $\frac{7}{12}$
7. $\frac{13}{17}$
8. $\frac{11}{19}$
9. $\frac{3}{4}, \frac{5}{7}, \frac{9}{13}$
10. $\frac{2}{5}, \frac{19}{48}, \frac{4}{11}$
11. $\frac{7}{15}, \frac{13}{28}, \frac{4}{9}$
12. $\frac{14}{17}, \frac{23}{28}, \frac{9}{11}, \frac{17}{21}$

SAMPLE PROBLEM 10

If $5 \le x \le 15$ and $20 \le y \le 30$, then which of the following intervals contains all possible values of $\frac{x}{y}$?

(A) from 5 to 30
(B) from $\frac{1}{4}$ to $\frac{3}{4}$
(C) from $\frac{1}{6}$ to $\frac{2}{3}$
(D) from $\frac{1}{6}$ to $\frac{7}{8}$
(E) from $\frac{1}{8}$ to $\frac{3}{8}$

Analysis

We must first note that the value of a fraction increases when the numerator increases and decreases when the denominator increases. Therefore, the smallest value of $\frac{x}{y}$ will be obtained when x has its smallest value, 5, and y has its largest value, 30. The smallest value of $\frac{x}{y}$ is, then, $\frac{5}{30}$ or $\frac{1}{6}$.

The largest value of $\frac{x}{y}$ will be obtained when x has its largest value, 15, and y has its smallest value, 20. The largest value of $\frac{x}{y}$ is $\frac{15}{20}$ or $\frac{3}{4}$.

The only interval among those listed that includes the full range from $\frac{1}{6}$ to $\frac{3}{4}$ is (D). It extends beyond the upper limit of $\frac{x}{y}$, but the wording of the question does not forbid this.

SAMPLE PROBLEM 11

If a solid ball 3" in diameter weighs 7 pounds, how many pounds are there in the weight of a ball of the same material 6" in diameter?

(A) 14 (B) 27 (C) 28 (D) 42 (E) 56

Analysis

We may assume the following relationships when comparing similar figures:

(ratio of corresponding linear parts)^2 = ratio of corresponding areas

(ratio of corresponding linear parts)^3 = ratio of corresponding volumes

Linear parts may be radii, circumferences, diameters, sides, edges, diagonals, perimeters, and so on.

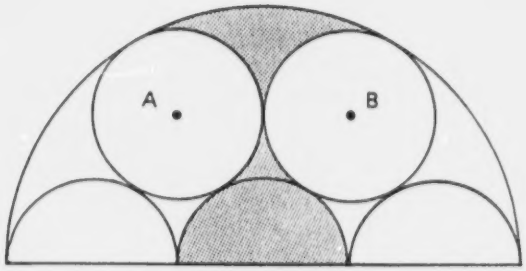
We employ the second relationship for this problem, since we are comparing volumes with diameters. (The weight of a solid is proportional to its volume.)

Since the ratio of the corresponding linear parts (diameters) is $\frac{1}{2}$, the ratio of the volumes is $\left(\frac{1}{2}\right)^3$ or $\frac{1}{8}$. Hence the weight of the second ball is 8 times the weight of the first, or 56 pounds.

The answer is (E).

SAMPLE PROBLEM 12

In the adjoining diagram, the two circles and three small semicircles are tangent to each other and to the large semicircle at their points of contact. The two circles and the three semicircles have equal radii of 1 unit each. Which of the following represents the area of the shaded region (in square units)?

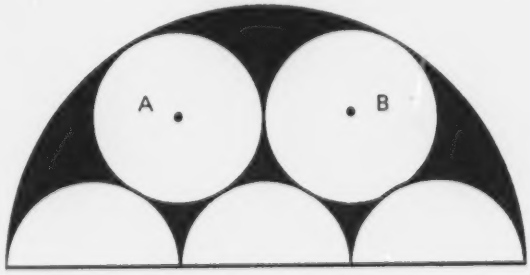


(A) $\frac{2\pi}{2}$ **(B)** $\frac{\pi}{2}$ **(C)** $\frac{5\pi}{6}$ **(D)** $\frac{7\pi}{6}$ **(E)** $\frac{3\pi}{2}$

Analysis

In SAT I area problems of this kind, you have to discover some geometric fact about the given diagram that simplifies the problem. In this case, you have to see that if the circles and semicircles are removed, the shaded area in the figure below is what remains. It consists of three congruent figures, one of which is part of the area we are trying to find. Therefore, the first step is to subtract the area of the two circles and three semicircles from the area of the large semicircle. Since the two circles are equivalent to four semicircles, the area to be subtracted is equal to seven semicircles of radius 1 unit. Since the area of the large semicircle is $\frac{1}{2}\pi(3)^2$ or $\frac{9}{2}\pi$, and the area of one small semicircle is $\frac{1}{2}\pi(1)^2$ or $\frac{\pi}{2}$, the area remaining is

$$\frac{9\pi}{2} - \frac{7\pi}{2} = \frac{2\pi}{2} = \pi.$$



The area we are interested in is $\frac{1}{3}$ of this, or $\frac{\pi}{3}$.

To this we must add the area of the shaded semicircle in the given figure. Hence the area of the shaded region is

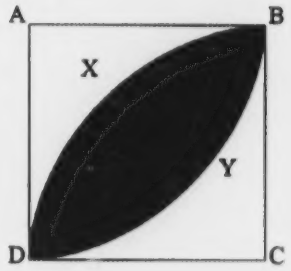
$$\frac{\pi}{3} + \frac{\pi}{2} = \frac{5\pi}{6}.$$

The answer is (C).

SAMPLE PROBLEM 13

In the adjoining square ABCD, arc DXB is part of a circle with center at C, and arc DYB is part of a circle with center at A. The area of square ABCD is 4 square units. Find the area between the arcs (i.e., the shaded region).

(A) π (B) $\pi - 2$ (C) $\pi - 4$ (D) $2\pi - 4$ (E) none of these

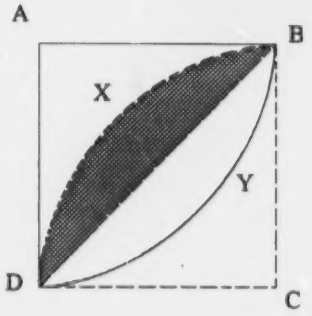


Analysis

Method 1

Consider the segment DXB of sector DXBC (shown by shading in the diagram below). If we find the area of this segment, then we have exactly one-half of the required area. To find the area of the segment, you must subtract the area of the triangle (formed by the radii $\overline{DC}$ and $\overline{BC}$ and the diagonal $\overline{BD}$ of the square) from the area of the sector. To find the area of sector DXBC we use the formula:

$$K = \frac{n}{360^0} \bullet \pi R^2 \text{ (n = the number of degrees in the arc)}$$



Since the area of the square is 4, each side is 2 (i.e., radius of sector = 2). Since all angles of a square are right angles, $n = 90^0$.

$$K = \frac{90^0}{360^0} \bullet \pi (2)^2$$

$$K = \frac{90 \bullet 4\pi}{360} = \pi$$

The area of right triangle BDC equals $\frac{1}{2} \bullet 2 \bullet 2 = 2$

Therefore, the area of the segment = $\pi - 2$.

However, we are required to find the area of the total shaded region, which equals twice the area of the segment; therefore, this area = $2\pi - 4$.

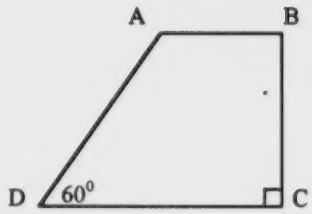
The answer is (D).

Method 2

(1) Add the areas of sectors ABD and CBD. The result is 2π.
(2) Subtract the area of square ABCD from this sum. The resulting area is that of BXDY, since the area of BXDY was used twice in the addition of the areas of the sectors. The result is $2\pi - 4$.

The answer is (D).

SAMPLE PROBLEM 14



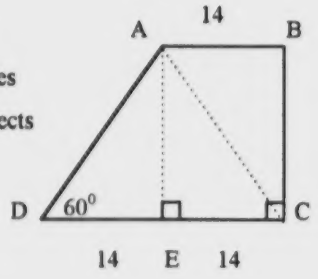
In trapezoid ABCD, $m\angle D = 60^0$, $m\angle C = 90^0$, AB = 14 inches.

Find the length of $\overline{AD}$ if AD = DC.

(A) 14 (B) $21\sqrt{3}$ (C) 21 (D) 28 (E) cannot be determined

Analysis

Since ΔADC is isosceles and contains an angle of measure 60^0, it is equilateral. Draw altitude $\overline{AE}$ to form rectangle ABCE. Since the lengths of the opposite sides of a rectangle are equal, AB = CE = 14. In an equilateral triangle, an altitude bisects the side to which it is drawn; therefore DE = EC = 14, and DC = 28.
Since AD = DC, AD = 28.
The answer is (D).



SAMPLE PROBLEM 15

Seven pounds of peaches cost as much as 1 pound of apples and 2 pounds of grapes. Seven pounds of apples cost as much as 10 pounds of grapes and 1 pound of peaches. How many pounds of grapes may be purchased for the amount of money required to purchase 12 pounds of apples?

(A) 9 (B) 12 (C) 18 (D) 24 (E) 28

Analysis

Let p = price of one pound of peaches
Let g = price of one pound of grapes
Let a = price of one pound of apples

From the first sentence we obtain the relationship:
$7p = a + 2g$ (1)
From the second sentence we obtain the relationship:
$7a = 10g + p$ (2)

Since there are three variables and only two equations, we know we cannot solve for any of the variables. But we can eliminate one variable and obtain a ratio between the other two. The problem does, in fact, require us only to find the ratio between g and a; so our objective is to eliminate p from Equations (1) and (2). There are various algebraic techniques for doing this. We can solve Equation (1) for p:

$$p = \frac{a + 2g}{7}$$

Then substitute this value in Equation (2).

We could, alternatively, multiply Equation (2) by 7 (to give us 7p in both equations) and then add the equations:

$$\begin{array}{r} 7p = a + 2g \\ 49a = 70g + 7p \\ \hline 7p + 49a = a + 2g + 70g + 7p \end{array}$$

By either method we obtain: $48a = 72g$.

This result tells us that 48 pounds of apples cost as much as 72 pounds of grapes. The problem asks for the equivalent of 12 pounds of apples. We can do this in routine fashion by solving for a in terms of g and multiplying by 12:

$$48a = 72g \qquad a = \frac{72g}{48} = \frac{3g}{2} \qquad 12a = 12 \bullet \frac{3g}{2} = 18g$$

We could also set up a proportion:

$$\frac{72g}{48a} = \frac{x}{12a}$$

which yields $x = 18g$.

Or best of all, we can see a shortcut and take it. Just divide by 4:

$48a = 72g$
$12a = 18g$

All methods give the same answer: (C).

SAMPLE PROBLEM 16

In checking petty cash, a clerk counts Q quarters, D dimes, N nickels, and P pennies. Later, he discovers that X of the nickels were counted as quarters and X of the dimes were counted as pennies. To correct the total obtained, the clerk must:

(A) make no correction (B) subtract 11¢ (C) subtract 11X¢ (D) add 11X¢ (E) add X¢

Analysis

The amount of cents first counted by the clerk was:

$25Q + 10D + 5N + P$ (1)

Later, he found that X nickels were counted as quarters. Therefore, he now has:

(Q - X) quarters
(N + X) nickels

He also found that X of the dimes were counted as pennies. Therefore, he now has:

(P - X) pennies
(D + X) dimes

The amount of cents the clerk actually has is therefore:

$25 (Q - X) + 5(N + X) + 10(D + X) + (P - X)$ (2)

This can be expanded to:

$25Q - 25X + 5N + 5X + 10D + 10X + P - X$
$= 25Q - 11X + 5N + 10D + P$ (3)

In order to determine the difference between his original count and his corrected count, we subtract line (1) from line (3) to obtain: $-11X$

This means that to obtain the true count from the original count, we must add $-11X¢$ to the original count. That is, we must subtract $11X¢$.

The answer is (C).

SAMPLE PROBLEM 17

Joyce, a champion long-distance runner, has 5 blue ribbons, 3 red ribbons, and 7 yellow ribbons in a drawer. If she is blindfolded, what is the least number of ribbons she must take from the drawer to be sure to get one of each color?

(A) 3 (B) 9 (C) 12 (D) 13 (E) 15

Analysis

We must assume that Joyce has the worst possible luck. The worst that can happen is that she takes 7 yellow ribbons and 5 blue ribbons for her first 12 selections. On her next selection, she must get a red ribbon, for there are no longer any blue or yellow ribbons left. Therefore, she is assured of getting one ribbon of each color on her 13^{th} selection.

The answer is (D).

SAMPLE PROBLEM 18

Joyce has 5 blue ribbons, 3 red ribbons, and 7 yellow ribbons in a drawer. If Joyce is blindfolded, what is the least number of ribbons she must take from the drawer to be sure to get two ribbons of the same color?

(A) 2 (B) 4 (C) 8 (D) 9 (E) 10

Analysis

We must again assume the worst possible luck. On her first 3 selections, she could take ribbons of three different colors. However, she cannot choose a different color on her fourth selection, since she already has one of each color. Hence she is assured of matching one of her previous selections on the fourth draw.
The answer is (B).

SAMPLE PROBLEM 19

Define a * b to equal 2a - b for all real numbers. Which of the following equals (5 * 3) * 1?

(A) 2 (B) 5 (C) 6 (D) 7 (E) 13

Analysis

Since a * b = 2a - b,

$5 * 3 = 2 \bullet 5 - 3 = 7$

Therefore:

$(5 * 3) * 1 = 7 * 1 = 2 \bullet 7 - 1 = 13$

The answer is (E).

Practice With Unfamiliar Operations

1. **Define $p \oplus q$ to equal $\frac{pq}{p - 2q}$ for all real numbers p and q, where $p \neq 2q$. What is the value of $(8 \oplus 3) \oplus 3$?**

2. **Define $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ to equal ad - bc for all real numbers.**

   For what value of x will $\begin{vmatrix} 2 & -3 \\ x & 5 \end{vmatrix} = 28$?

Answers: 1. 1.6 2. 2.6

SAMPLE PROBLEM 20

Two fixed points, x and y, are located on a straight line 100 feet apart. A movable point P moves from x to y in the following uniform manner: During the first half of each minute P moves 3 feet toward y, and during the second half of each minute it returns 2 feet toward x. In how many minutes will P reach y?

(A) 100 min. (B) $99\frac{1}{2}$ min. (C) $98\frac{1}{2}$ min. (D) 98 min. (E) $97\frac{1}{2}$ min.

Analysis

It should be clear that after 1 complete minute has elapsed, the point P has progressed 1 foot toward y. After 2 complete minutes it has progressed 2 feet. After 10 complete minutes it has progressed 10 feet. By the same reasoning, we find that after 97 complete minutes, point P has progressed 97 feet.
However, during the first half of the 98th minute, it will travel 3 feet beyond the 97-foot mark; in other words, it will reach the 100-foot mark in that time interval. Therefore, point P has traveled 100 feet in $97\frac{1}{2}$ minutes.

The answer is (E).

SAMPLE PROBLEM 21

If $16\frac{2}{3}\%$ of the cars sold by a car dealer are Brand X, and this amount is $66\frac{2}{3}\%$ of the sales of Brand Y, then what percent of the entire business is the sale of the other cars?

(A) $6\frac{2}{3}\%$ (B) 25% (C) $58\frac{1}{3}$ (D) $83\frac{1}{3}\%$ (E) 91%

Analysis

To avoid working with cumbersome arithmetic, we will change each of the percents to fractions:

$$16\frac{2}{3}\% = \frac{1}{6}$$

$$66\frac{2}{3}\% = \frac{2}{3}$$

Let p = part of cars sold which are Brand Y.

$$\frac{2}{3}p = \frac{1}{6}$$

$$p = \frac{1}{4}$$

Thus we have found:

$\frac{1}{6}$ of the cars sold are Brand X

$\frac{1}{4}$ of the cars sold are Brand Y

Therefore, by addition:

$\frac{5}{12}$ of the cars sold are Brand X plus Brand Y, and $\frac{7}{12}$ of cars sold are "other cars": $\frac{7}{12} = 58\frac{1}{3}\%$

The answer is (C).

SAMPLE PROBLEM 22

Next year, the number of students in elementary schools is expected to be $\frac{1}{5}$ of the population; there will be $\frac{1}{4}$ as many students in high schools as in elementary schools, and $\frac{2}{5}$ as many in colleges as in high schools. What percent of the population is expected to be in colleges?

(A) 2% (B) 4% (C) 8% (D) 10% (E) 15%

Analysis

In a problem involving a chain of relationships, we may begin at either end of the chain. By using the initial letters of the words elementary, high (school), and college to represent their numbers, and p to represent the entire population, we have:

$$e = \frac{1}{5}p; \quad h = \frac{1}{4}e; \quad c = \frac{2}{5}h$$

$$c = \frac{2}{5}h = \frac{2}{5}\left(\frac{1}{4}e\right) = \frac{1}{10}\left(\frac{1}{5}p\right) = \frac{1}{50}p$$

The desired relationship is: $c = \frac{1}{50}$ p, or .02p, or 2% of p

The answer is (A).

SAMPLE PROBLEM 23

If the selling price of a jacket is $7.20 when the merchant makes a 20% profit on the cost, how much would the merchant have to raise the selling price m order to increase her profit to 30%?

(A) $.30 (B) $1.80 (C) $.20 (D) $.60 (E) $.40

Analysis

The selling price = cost + profit. If the profit is 20% of the cost, then:

selling price = cost + 20% of cost

If we want the new selling price to include a 30% profit, then:

new selling price = cost + 30% of cost

We can see, therefore, that the selling price must be increased by an amount equal to 10% of the cost. Our problem, then, is to find the cost.

Let C = cost. Then:

$C + .20C = \$7.20$ $\qquad C(1 + .20) = \$7.20$ $\qquad C = \frac{\$7.20}{1.20} = \$6.00$

10% of C = .10($6.00) = $ .60

The answer is (D).

SAMPLE PROBLEM 24

Since Henry received a speeding ticket on his way to work one day, he decided to cut his speed by $16\frac{2}{3}\%$. Which of the following represents the increase in time required for Henry to get to work?

(A) $16\frac{2}{3}\%$ **(B)** 20% **(C)** $22\frac{1}{2}\%$ **(D)** 80% **(E)** $83\frac{1}{3}\%$

Analysis

Let r = Henry's original rate Let t = Henry's original time Let d = original distance

We may establish the following relationship:

$r\,t = d$

Since Henry's rate is being decreased by $\frac{1}{6}$ (note: $16\frac{2}{3}\% = \frac{1}{6}$), his new rate is $\frac{5}{6}$ his original rate. Therefore, his new time must be $\frac{6}{5}$ his original time, since $(\frac{5}{6}r)(\frac{6}{5}t) = d$. However, $\frac{6}{5}t$ represents an increase of t by $\frac{1}{5}$, or a 20% increase in time.

The answer is (B).

SAMPLE PROBLEM 25

An airplane flew from A to B, a distance of 450 miles, at an average speed of 180 miles per hour. The return journey was made at an average speed of 90 miles per hour. What was the average speed of the whole journey in miles per hour?

(A) 135 (B) 127.5 (C) 125 (D) 120 (E) none of these

Analysis

We must first understand what is meant by "average speed." In problems involving travel at different speeds, it means the single speed which, if maintained throughout the journey, would accomplish the trip in the same total time. For example, if a trip of 120 miles requires 3 hours, the average speed is 40 miles per hour, regardless of how the speed varies during the trip. A little experimentation will show that you can average different speeds if they are maintained for equal periods of time, but not if they are maintained for equal distances. There are various ways of calculating the average speed in problems of the kind presented here.

Method 1

Let us use the relationship

$$\text{Rate} \times \text{Time} = \text{Distance, or Time} = \frac{\text{Distance}}{\text{Rate}}$$

to calculate the time for each part of the trip; once we find the total time, we can calculate the average rate over the total distance:

$$T_1 = \frac{D_1}{R_1} = \frac{450 \text{ miles}}{180 \frac{\text{miles}}{\text{hour}}} = 2\frac{1}{2} \text{ hours} \qquad T_2 = \frac{D_2}{R_2} = \frac{450 \text{ miles}}{90 \frac{\text{miles}}{\text{hour}}} = 5 \text{ hours}$$

The total time T for the total distance D of 900 miles is $7\frac{1}{2}$ hours.

The average rate R is:

$$R = \frac{D}{T} = \frac{900 \text{ miles}}{7\frac{1}{2} \text{ hours}} = 120 \text{ miles per hour}$$

The answer is (D).

Method 2

If you like to work with general formulas, instead of working out the arithmetic for each problem, the following approach will appeal to you. Let us find the average rate R for any round trip which is made one way at rate R_1, and the other way at rate R_2. As in Method 1:

$$T_1 = \frac{d}{R_1} \qquad T_2 = \frac{d}{R_2} \qquad \text{(where d represents the distance going one way)}$$

$$T = T_1 + T_2 = \frac{d}{R_1} + \frac{d}{R_2} = d\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Since the total distance is 2d:

$$R = \frac{2d}{T} = \frac{2d}{d\left(\frac{1}{R_1} + \frac{1}{R_2}\right)} = \frac{2}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{2R_1R_2}{R_1 + R_2}$$

Notice that this expression is the reciprocal of the average of the reciprocals of the two rates. That is, if we take the reciprocals of R_1 and R_2, then find the average of these reciprocals, and then take the reciprocal of this average, we will have the average rate. This quantity is called the HARMONIC MEAN. The harmonic mean of two quantities a and b is:

$$\frac{1}{\frac{\frac{1}{a} + \frac{1}{b}}{2}} = \frac{2ab}{a + b}$$

The average rate when half a trip is made at one speed and the other half at another speed is the harmonic mean of the two speeds. In this problem:

$$R = \frac{2 \cdot 90 \cdot 180}{90 + 180} = 120$$

Method 3

The idea behind the harmonic mean method is that the time for a given trip is inversely proportional to the rate. Therefore, in finding the average rate, the slower speed must be given proportionately more "weight." In this problem, we can see that for each hour spent traveling at 180 miles per hour, 2 hours must be spent at 90 miles per hour to cover the same distance. Therefore, out of every 3 hours, 2 are spent at 90 miles per hour and 1 at 180 miles per hour. We can find the average, then, by adding 90 + 90 + 180 and dividing by 3:

$$\frac{90 + 90 + 180}{3} = \frac{360}{3} = 120$$

This is somewhat of an intuitive method and should not be risked unless you fully understand the principles behind it. But if you can use it, it is the fastest of the three approaches.

Practice With Uniform Motion Problems

1. A small plane left an airport, heading north. A helicopter, also heading north, was 140 miles north of the airport. The plane traveled at 150 miles per hour and overtook the helicopter in 2 hours and 20 minutes. What was the average speed of the helicopter?

2. Carol drove 225 miles at 25 miles per hour. The return trip took her 15 hours. What was her average speed for the trip?

3. Charlotte took a 175-mile trip, traveling 10 miles per hour faster on the way there than on the return trip. If the return trip was 2 hours longer, what was her speed in miles per hour on the way to her destination?

4. Paul traveled a certain distance at 20 miles an hour and the same distance at 25 miles an hour. He spent a total of $4\frac{1}{2}$ hours in making the trip. Find the distance.

5. Larry starts on a bicycle $1\frac{1}{2}$ hours after his sister, who rides at the rate of 8 miles per hour. He overtakes her in 6 hours. At what rate per hour did he ride?

6. An eagle-eyed traffic police officer saw a speeding vehicle $\frac{1}{2}$ mile away and, pursuing it at 90 mph, overtook it in 3 minutes. What was the vehicle's speed?

Answers: 1. 90 mph 2. $18\frac{3}{4}$ mph 3. 35 mph 4. 50 mi 5. 10 mph 6. 80 mph

SAMPLE PROBLEM 26

A master cobbler can complete a job in 5 days when he works alone. With the help of his apprentice, the job can be completed in 3 days. How long would it take the apprentice to do the job alone?

(A) $6\frac{1}{2}$ **days** **(B) 7 days** **(C)** $7\frac{1}{2}$ **days** **(D) 8 days** **(E)** $8\frac{1}{2}$ **days**

Analysis

If a master cobbler can complete a job in 5 days working alone, then he can do $\frac{1}{5}$ of the job in one day. Similarly, the cobbler and his apprentice can complete $\frac{1}{3}$ of the job in one day. If x = the number of days it would take the apprentice to complete the job working alone, then $\frac{1}{x}$ of the job can be completed in one day by the apprentice working alone. (Note: Whenever possible, let your variable (x) represent the quantity that the question asks for.)

Method 1

The part of the job the master can do in one day	+	The part of the job the apprentice can do in one day	=	The part of the job they can do together in one day

That is,

$\frac{1}{5} + \frac{1}{x} = \frac{1}{3}$ $\qquad x = 7\frac{1}{2}$

The answer is (C).

Method 2

Since the master cobbler can do $\frac{1}{5}$ of the job in 1 day, he can do $\frac{3}{5}$ of the job in 3 days. Similarly, since the apprentice can do $\frac{1}{x}$ of the job in 1 day, he can do $\frac{3}{x}$ of the job in 3 days. However, since the job would be completed in 3 days if they worked together, we may conclude that:

$\frac{3}{5} + \frac{3}{x} = 1$ (1 represents the entire job) $\quad x = 7\frac{1}{2}$

Practice With Work Problems (Unequal Rates of Work)

1. Barbara completed $\frac{2}{3}$ of a job in 8 hours. She was then joined by Al. Together they finished the job 3 hours later. How many hours would it have taken Al to do the job alone?

2. Working alone, pipe A can fill an entire tank in 40 minutes. Working alone, pipe B can fill the same tank in 80 minutes. However, the drainpipe can empty the entire tank in 2 hours. If both pipes A and B are filling the tank while the drainpipe is emptying the tank, how many minutes will it take the tank to fill to capacity?

Answers: 1. 36 2. $34\frac{2}{7}$

SAMPLE PROBLEM 27

Three contractors supplied laborers for a job. The first contractor supplied 5 people for 3 days; the second, 6 people for 7 days; the third, 8 people for 6 days. The contractors paid the laborers a total of $3150 in wages. If each laborer received the same daily wage, how much did the second group of laborers receive for their part of the job?

(A) $983.33 (B) $987.40 (C) $1260 (D) $1290 (E) none of these

Analysis

We can answer this question if we know how much each person is paid for 1 day's work. We may call this amount of work 1 "man-day" of work. By dividing the total amount of wages by the total number of man-days in the whole job, we can find the cost of a single man-day:

The first group consists of 5 laborers working 3 days, yielding 15 man-days.

The second group contributes 6 X 7 = 42 man-days.

The third group contributes 8 X 6 = 48 man-days.

The total job = 15 + 42 + 48 = 105 man-days.

Since the contractors paid a total of $3150 for 105 man-days, the wage for 1 man-day is:

$$\frac{\$3150}{105} = \$30$$

The second group, which worked 42 man-days, therefore received:

$$42 \cdot \$30 = \$1260$$

The answer is (C).

Note: The principle of the "man-day" of work (or "man-hour," or "man-year," or any other unit of work) can be applied to any work problem in which everyone works at the same rate. It does not apply to problems like those in the preceding group, where the rates of work are unequal.

Practice With Work Problems (Equal Rates of Work)

Hint: In each of these problems, express the total job as a product of the number of people and the time they work. For the same job, this total remains the same.

1. If X gardeners can mow a lawn in Y hours, how long would it take Z gardeners to mow the same lawn?

2. Twelve ranchers can put up 100 feet of fencing in 6 hours. If 3 ranchers did not work, how long would it take the remaining number of ranchers to put up 100 feet of the same fencing?

3. If 10 cans of food are needed for 6 campers for 2 days, how many cans of food are required for 8 campers for 3 days?

4. If 13 loggers can cut 364 logs of wood in 14 days, working 12 hours per day, how many hours a day must 15 loggers work to cut 810 logs of wood in 36 days?

Answers: 1. XY/Z 2. 8 hours 3. 20 4. 9

SAMPLE PROBLEM 28

A man buying a coat is faced with a decision. The merchant offers him two successive discounts of 20% and 10% or one of 29%. The difference between these two offerings is:

(A) 3% (B) 2% (C) 1% (D) 0 (E) none of these

Analysis

Method 1

There is a mechanical method for obtaining a single percentage discount (or increase) equivalent to two (or more) successive discounts (or increases).

(1) Change each of the percentages involved into decimal form:

.20 and .10

(2) Subtract each of these decimals from 1.00:

.80 and .90 (for an increase, add to 1.00)

(3) Multiply these differences:

(.80) (.90) = .72

(4) Subtract this number (i.e., .72) from 1.00:

1.00 - .72 = .28 (if the result of Step 3 is greater than 1.00, subtract 1.00 from it to obtain the percent of *increase*)

When we convert .28 back to percent form, we obtain 28%, the equivalent of successive discounts of 20% and 10%. This percentage (i.e., 28%) differs from 29% by 1%.

The answer is (C).

Note: We may see from this method of solution that the order in which the successive discounts are taken is unimportant in the solution of the problem.

Method 2

Some students may feel more sure of the result by assuming an initial amount of $100 and then applying the given discounts to see what the net reduction amounts to. However, this takes more time and usually requires more computation. Despite using a calculator, sometimes more calculations increase the chance for an error. In this case, 20% discount off $100 is $80; then 10% discount off $80 is $72. This represents a 28% discount, which differs from the 29% discount by 1%. Readers are urged to practice Method 1 until they have mastered it.

Practice With Successive Discounts

1. What is the difference between successive discounts of 10%, 30%, and 50%, and one discount of 65%?

2. Article A is discounted by 10%, and 10%, and then has its price raised by 20%. Article B, which has the same original price as article A, has its price discounted by 10% and then raised by 10%. How do the final prices compare?

3. When the entrance price to a baseball game was decreased by 25%, the attendance at the game rose by 30%. What was the effect of these changes on the daily receipts?

Answers: 1. successive discounts are $3\frac{1}{2}\%$ greater 2. A is 1.8% less than B 3. $2\frac{1}{2}\%$ decrease

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SAMPLE PROBLEM 29

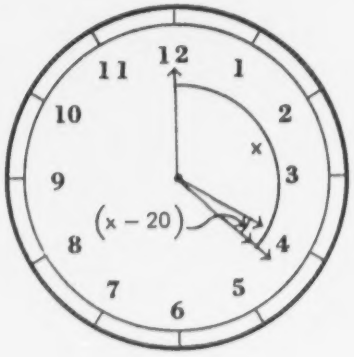
At what time after 4:00 will the minute hand overtake the hour hand?

(A) 4:20 (B) 4:21 (C) 4:21 $\frac{7}{11}$ (D) 4:21 $\frac{9}{11}$ (E) 4:22

Analysis

We may deal with this problem as we would if asked to find the time it took a fast car to overtake a slower one.

Let us speak of the rate (i.e., speed) of the hour hand as r. Then the rate of the minute hand is 12r. The distance that the hands travel will be measured by the minute markers on the clock. What we seek in this problem is the distance (in minutes) that the minute hand must travel to overtake the hour hand. Let this distance be "x." Therefore, the distance that the hour hand must travel is x - 20, since it has a 20-minute head start over the minute hand.



Since the time equals the distance divided by the rate, the time that the minute hand travels is $\frac{x}{12r}$, and the time that the hour hand travels is $\frac{x - 20}{r}$.

Since both hands travel the same amount of time:

$$\frac{x}{12r} = \frac{x - 20}{r}$$

$$x = 12(x - 20) = 12x - 12 \cdot 20$$

$$11x = 12 \cdot 20 \qquad \text{(see note below)}$$

$$x = \frac{12}{11} \cdot 20$$

$$x = 21\frac{9}{11}$$

Therefore, at $4{:}21\frac{9}{11}$ the minute hand will overtake the hour hand.

The answer is (D).

Note: In the relation $x = \frac{12}{11} \cdot 20$ the quantity 20 is the number of minutes of head start that the hour hand has over the minute hand. The ratio $\frac{12}{11}$, then, is the number of minutes required per minute of head start for the minute hand to overtake the hour hand. Therefore, we can substitute any known value for the 20 and find the time required by multiplying by $\frac{12}{11}$.

For example, if the time at the start is 8:00,

$$x = \frac{12}{11} \cdot 40 = 43\frac{7}{11}$$

the minute hand will overtake the hour hand at $8{:}43\frac{7}{11}$.

Another way of looking at this relation is to say that the head start of 20 minutes (or whatever the amount) is the time it would take the minute hand to reach the hour hand if the hour hand did not move. But since the hour hand does move, it takes $\frac{12}{11}$ as long. This idea can be applied to many variations of the clock problem.

SAMPLE PROBLEM 30

At what time after 7:30 will the hands of a clock be perpendicular?

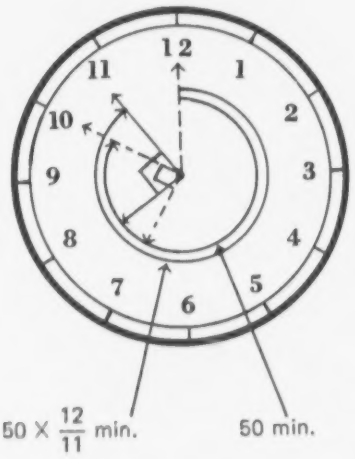
(A) 7:20 (B) 7:50 (C) $7{:}54\frac{6}{11}$ **(D)** $7{:}55\frac{5}{11}$ **(E)** $7{:}53\frac{7}{11}$

Analysis

Let us assume that the hour hand does not move after 7:00. Then the minute hand would be perpendicular to the hour hand at 7:50. (This would happen at 7:20 also, but the problem asks for a time after 7:30.) The travel time for the minute hand, with a stationary hour hand, is 50 minutes. With a moving hour hand, it must be $\frac{12}{11}$ as much:

$$\frac{12}{11} \cdot 50 = 54\frac{6}{11}$$

The hands will be perpendicular at $7{:}54\frac{6}{11}$.



If you think you understand the basic principle, try to solve the following problem:

Between 3:00 and 4:00, Grace looked at her watch and noticed that the minute hand was between "5" and "6." Later, Grace looked again and noticed that the hour hand and the minute hand had exchanged places. What time was it in the second case?

SOLUTION:

Let x = position of minute hand between 3 and 4 o'clock; then the position of hour hand is $(15 + \frac{x}{12})$.

Let y = position of minute hand between 5 and 6 o'clock; then the position of hour hand is $(25 + \frac{x}{12})$.

Since hands have changed places, $15 + \frac{x}{12} = y$ and $25 + \frac{y}{12} = x$. Solve for y: $y = 17\frac{29}{143}$.

The answer is $5{:}17\frac{29}{143}$.

III. Sample Quantitative Comparisons

Each question of the Quantitative Comparison Section asks you to compare two quantities, one in Column A and the other in Column B. To answer these questions you must be familiar with the concepts of greater than (>), less than (<), and equal to (=) and with their application to arithmetic, algebra, and geometry. Perhaps a review of such applications will prove helpful.

These questions generally require less time to answer than the multiple-choice questions. However, to reduce further the time consumed in answering the quantitative comparison questions, you are advised to familiarize yourself thoroughly with the instructions and the significance of the four choices: A, B, C, and D. The instructions and notes are reprinted here exactly as they will appear on the actual *SAT-I*.

> ***Directions: Each question in this section consists of two quantities, one in column A and one in Column B. You are to compare the two quantities and on the answer sheet blacken space**
>
> **A if the quantity in Column A is the greater**
> **B if the quantity in Column B is the greater**
> **C if the two quantities are equal**
> **D if the relationship cannot be determined from the information given**
>
> **Notes: (1) In certain questions, information concerning one or both of the quantities to be compared is centered above the two columns. (2) A symbol that appears in both columns represents the same thing in Column A as it does in Column B. (3) All numbers in this test are real numbers. Letters such as x, n, and k stand for real numbers.**

Each sample problem presents an important concept often used in answering the quantitative comparison questions. Special attention should be given to these sample problems before attempting the four Quantitative Comparison Practice Tests (pages 118-134).

SAMPLE PROBLEM 1

Column A		Column B
	$-7x > 35$	
x		-5

Analysis

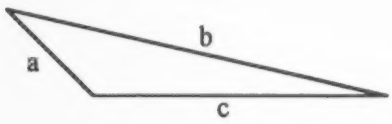
To solve the given inequality, multiply both sides by $-\frac{1}{7}$ to get $x < -5$. Remember that when multiplying (or dividing) both sides of an inequality by a negative number, the order of the inequality must be changed.

The answer is (B).

ANSWER CHOICES

A if the quantity in Column A is the greater
B if the quantity in Column B is the greater
C if the two quantities are equal
D if the relationship cannot be determined from the information given

SAMPLE PROBLEM 2

Column A		Column B
b		a - c

Analysis

In every triangle the sum of the lengths of any two sides is greater than the length of the remaining side. Here that relationship yields $b + c > a$. By subtracting c from both sides of the inequality we get $b > a - c$.

The answer is (A).

SAMPLE PROBLEM 3

Column A	$x^2 + 6x + 8 = 0$	Column B
x		0

Analysis

Since $x^2 + 8$ must be positive and $x^2 + 6x + 8$ is 0, it follows that $6x$ must be negative, or $6x < 0$. Therefore $x < 0$.

An *alternate method* of comparing x and 0 involves solving the equation $x^2 + 6x + 8 = 0$

Factor the trinomial: $(x + 2)(x + 4) = 0$

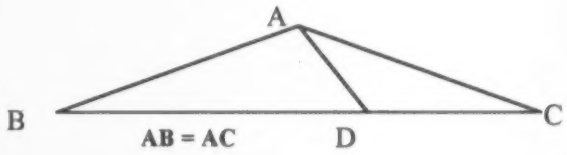
Let each factor equal 0: $x + 2 = 0$ $\quad x + 4 = 0$

$x = -2$ $\quad x = -4$

In both cases, $x < 0$.

The answer is (B).

SAMPLE PROBLEM 4

Column A		Column B
AB	 AB = AC	AD

Analysis

The measure of an exterior angle of a triangle is greater than the measure of each remote interior angle. Therefore, in ΔADC we have $m\angle ADB > m\angle C$. However, since ΔABC is isosceles (AB = AC), we have $m\angle C = m\angle B$. By substitution we conclude that $m\angle ADB > m\angle B$.

If the measures of two angles of a triangle are unequal, then the lengths of the sides opposite these angles are unequal, with the longer side being opposite the angle with the greater measure. Therefore, in ΔABD, AB > AD.

The answer is (A).

SAMPLE PROBLEM 5

Column A	Column B
The measure of the obtuse angle formed by the hands of a clock at 4:40.	**The measure of the obtuse angle formed by the hands of a clock at 5:45.**

Analysis

The degree measure of each interval between consecutive numbers on the face of a clock is 30^0. The measure of the angle formed by the hands of a clock at 4:40 is not 120^0 (i.e., $4 \cdot 30^0$), since the hour hand is *not* pointing to the 4. At 4:40 the hour hand is located $\frac{40}{60}\left(\text{or } \frac{2}{3}\right)$ the distance from the 4 to the 5. Therefore, the measure of the angle formed by the hands of a clock at 4:40 is $(\frac{1}{3})\,(30) + 3\,(30) = 100$, that is, one third of the interval between 4 and 5, plus the three intervals between 5 and 8.

The measure of the angle formed by the hands of a clock at 5:45 may be found similarly. At 5:45 the hour hand is situated $\frac{45}{60}\left(\text{or } \frac{3}{4}\right)$ the distance from the 5 to the 6. Therefore, the measure of the angle formed by the hands of the clock at 5:45 is $(\frac{1}{4})(30) + 3(30) = 97\frac{1}{2}$, that is, one fourth of the interval between 5 and 6, plus the three intervals between 6 and 9.

The answer is (A).

ANSWER CHOICES

A if the quantity in Column A is the greater
B if the quantity in Column B is the greater
C if the two quantities are equal
D if the relationship cannot be determined from the information given

SAMPLE PROBLEM 6

Column A		Column B
	$x > y$	
$x + \frac{y}{2}$		$y + \frac{x}{2}$

Analysis

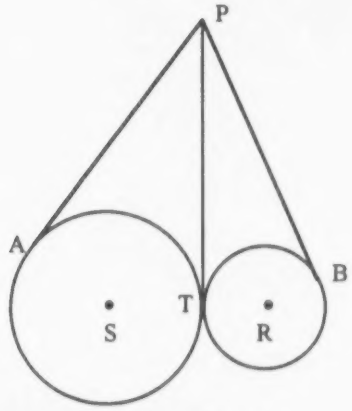
If $x + y$ is added to both sides of the given inequality $x > y$, this inequality results: $2x + y > 2y + x$.

Dividing both sides by 2 yields: $x + \frac{y}{2} > y + \frac{x}{2}$.

The answer is (A).

SAMPLE PROBLEM 7

Column A	Column B



$\overline{PA}$ is tangent to circle S at Point A.
$\overline{PB}$ is tangent to circle R at point B.
$\overline{PT}$ is tangent to both circles at point T.

PA	PB

Analysis

Two tangent segments to the same circle from the same external point have the same length. Therefore, PA = PT and PT = PB. Hence PA = PB.

The answer is (C).

SAMPLE PROBLEM 8

Column A		Column B
	$an > bn$	
a		b

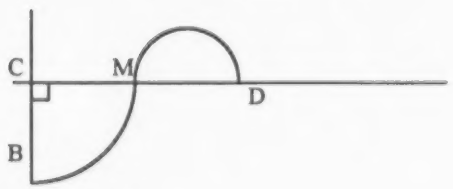
Analysis

The relationship between a and b cannot be determined because we do not know whether n is positive or negative. (If $n > 0$, then $a > b$; but if $n < 0$, then $a < b$.) It is important to remember that when multiplying or dividing both sides of an inequality by a negative number, the order of the inequality must be changed.

The answer is (D).

SAMPLE PROBLEM 9

Column A



$\overline{CB} \perp \overline{CD}$
$CM = DM$
$\overline{BM}$ and $\overline{DM}$ are arcs of circles with centers on $\overline{CMD}$.

Column B

length of BM		length of DM

Analysis

The arc length of *quarter circle* BM is twice as long as the arc length of one-half *semicircle* DM, since the radius of the former is twice as long as the radius of the latter. Therefore, the arc length of quarter circle BM equals the arc length of semicircle DM.

An *alternate method* of solution is to let CM = r and DM = r. Then the arc length of quarter circle BM $= \frac{1}{4}(2\pi r) = \frac{\pi r}{2}$, and the arc length of semicircle DM (whose radius is $\frac{r}{2}$) $= \frac{1}{2}(2\pi \frac{r}{2}) = \frac{\pi r}{2}$. Thus both arcs have the same length.

The answer is (C).

ANSWER CHOICES

A if the quantity in Column A is the greater
B if the quantity in Column B is the greater
C if the two quantities are equal
D if the relationship cannot be determined from the information given

SAMPLE PROBLEM 10

Column A	Column B
The number of minutes in a week	**The number of seconds in 7 hours**

Analysis

It is not necessary to do any arithmetic computation to answer this question. Simply set up the necessary computation and inspect it.

The number of minutes in a week can be found by multiplying $60 \cdot 24 \cdot 7$, since there are 60 minutes in an hour, 24 hours in a day, and 7 days in a week.

The number of seconds in 7 hours can be found by multiplying $60 \cdot 60 \cdot 7$, since there are 60 seconds in a minute, 60 minutes in an hour, and there are 7 hours considered.

By inspection, $60 \cdot 60 \cdot 7 > 60 \cdot 24 \cdot 7$.

The answer is (B).

IV. Practice Tests

Be sure to time yourself on each test. Do not be alarmed if you do not finish a test in the allotted time; these practice tests, like the *SAT-I* itself, are planned to be too long for all but a very few top students. When your time is up on a practice test, mark the point you have reached; then go on and finish the test. Try to beat your record on each succeeding test.

Answers to the practice tests are on page 205. Solutions begin on page 142. Do not check the answers or read the solutions until after you have completed the test.

A cross-index to the practice test problems, arranged by topic, begins on page 202. You may use this index to review problems based on specific areas of mathematics, but it would be wise to do the tests in their entirety before using the index for such special review.

Multiple-Choice Problems

MATHEMATICS TEST 1
Time Limit: 30 Minutes

1. If $\frac{1}{y} = \frac{3}{5}$, then y = ?

   (A) $\frac{3}{5}$ (B) $\frac{5}{3}$ (C) 5 (D) 3 (E) 15

2. $\dfrac{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}}{\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}} = ?$

   (A) $\frac{3}{5}$ (B) $\frac{35}{24}$ (C) $\frac{1}{15}$ (D) $1\frac{2}{3}$ (E) $1\frac{1}{3}$

3. If x = 5y - 2, then 3x - 4 = ?

   (A) 15y + 2 (B) 15y - 2 (C) 15y - 3 (D) 15y + 10 (E) 15y - 10

4. If xyz = 1 and x = z, then y = ?

   (A) $1 - x^2$ (B) x^2 (C) 1 - 2x (D) $\frac{1}{2x}$ (E) $\frac{1}{x^2}$

5. If $\frac{n}{5} > n$, then n may take on which of the following values?

   (A) -20 (B) 5 (C) 20 (D) 0 (E) none of these

6. If $a + \frac{1}{a} = b + \frac{1}{b}$, then a equals which of the following?

   I. b II. -b III. $\frac{-1}{b}$ IV. $\frac{1}{b}$

   (A) I or IV (B) II or III (C) I only (D) III only (E) III or IV

7. If the taxi rates are 25¢ for the first mile and 5¢ for each additional mile, and if Carol hires a taxi for p miles, what is the fare in cents? (Assume p is an integer ≥1)

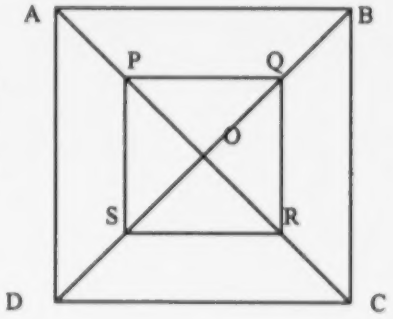
   (A) 25 + p (B) 20 - 5p (C) 20 + 5p (D) 25 + 5p (E) 25p + 5

8. Barbara has a closet filled with jackets. She has eight green jackets, five white jackets, three red jackets, and two blue jackets. Since the room is dark, she cannot see the color of each jacket. She would like to have two jackets of the same color. How many jackets does she have to take until she is assured of having at least two jackets of the same color?

(A) 3 (B) 4 (C) 5 (D) 6 (E) 8

9. In the adjoining diagram, square ABCD has diagonals $\overline{AC}$ and $\overline{BD}$ intersecting at O. PQRS is a square formed by joining the midpoints of $\overline{AO}$, $\overline{BO}$, $\overline{CO}$, and $\overline{DO}$, respectively. What is the ratio of the area of PQRS to that of ABCD?

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{1}{16}$ (D) $\frac{1}{4}$ (E) $\frac{1}{9}$



10. If the average of four numbers is 8, and three of the four numbers are 3, 5, and 9, find the fourth number.

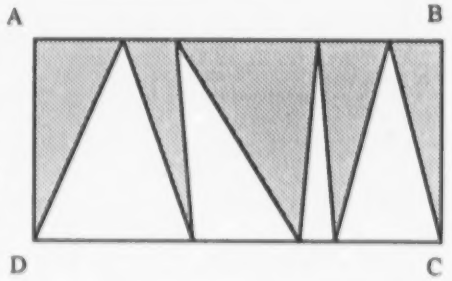
(A) 12 (B) 13 (C) 14 (D) 15 (E) 16

11. A long-playing phonograph record turns at the rate of $33\frac{1}{3}$ revolutions per minute. In how many seconds will the record revolve 180^0?

(A) $\frac{1}{3}$ (B) 3 (C) $1\frac{4}{5}$ (D) $\frac{9}{10}$ (E) 10

12. ABCD is a rectangle; the relationship of $\frac{\text{area of shaded region}}{\text{area of rectangle}}$ is

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$ (E) cannot be determined



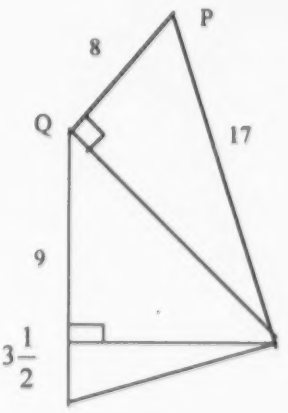
13. Find the three missing numbers in the following series:

1, 3, 2, 6, 5, 15, 14, __, __, __, 122, . . .

(A) 42, 41, 123 (B) 13, 39, 38 (C) 24, 23, 123
(D) 28, 27, 121 (E) 13, 39, 123

14. In this diagram, $m\angle PQT$ and $m\angle TRS$ are each 90^0. PT = 17. PQ = 8. RQ = 9. RS = $3\frac{1}{2}$. Find ST.

(A) 12 (B) $12\frac{1}{2}$ (C) 15 (D) 25 (E) none of these



15. A Danish krone is worth 50 French francs; an Egyptian pound is worth $17\frac{1}{2}$ krone; there are 350 French francs to \$1.00. How many Egyptian pounds are equivalent to \$1.00?

(A) .4 (B) 4 (C) $2\frac{1}{2}$ (D) $\frac{1}{4}$ (E) none of these

16. In a department consisting of three Divisions, Division A has 900 employees, Divisio has 500, and Division C has 600. If each division is to be reduced proportionately until there are 1500 employees in the department, how many employees will remain in Division C?

(A) 400 (B) 450 (C) 480 (D) 500 (E) 540

17. Richie did $\frac{2}{5}$ of a job in $\frac{1}{5}$ of an hour. At that rate, how long would it take him to do the whole job?

(A) 1 hour (B) $\frac{1}{5}$ hour (C) $\frac{1}{2}$ hour (D) $\frac{2}{5}$ hour (E) $\frac{1}{3}$ hour

18. A rectangular sheet of paper has an area of 24 square inches. If 4 holes, each having a radius of $\frac{1}{2}$ inch, are punched in this sheet of paper, what percent of the area of the rectangle has been punched out?

(A) $\frac{\pi}{2400}$ (B) $\frac{24 - \pi}{2400}$ (C) $\frac{25\pi}{6}$ (D) $\frac{50(6 - \pi)}{3}$ (E) $\frac{25(24 - \pi)}{6}$

19. If 2 people working 2 hours a day for 2 days produce 2 articles, then how many articles would 6 people produce working 6 hours a day for 6 days?

(A) 6 (B) 18 (C) 24 (D) 36 (E) 54

20. To win an election, Edith needs $\frac{3}{4}$ of the votes cast. If after $\frac{2}{3}$ of the votes have been counted Edith has $\frac{5}{6}$ of what she needs, what part of the remaining votes does she still need?

(A) $\frac{1}{8}$ (B) $\frac{3}{8}$ (C) $\frac{5}{8}$ (D) $\frac{1}{24}$ (E) $\frac{5}{24}$

MATHEMATICS TEST 2
Time Limit: 30 Minutes

1. Which of the following fractions is greater than $\frac{1}{4}$ but less than $\frac{1}{3}$?

   (A) $\frac{11}{32}$ (B) $\frac{4}{13}$ (C) $\frac{3}{16}$ (D) $\frac{6}{17}$ (E) $\frac{6}{25}$

2. If x is between 0 and 1, which of the following increases as x increases?

   (A) $\frac{1}{x}$ (B) $1 - x^2$ (C) $x - 1$ (D) $1 - x$ (E) $1\frac{1}{x^2}$

3. If the area of a circle is numerically equal to the circumference of the same circle, then the radius is

   (A) 2 (B) 3 (C) 4 (D) 5 (E) π

4. If $x + y = 1$ and $\frac{x}{y} = 1$, then $xy = ?$

   (A) 1 (B) $\frac{1}{4}$ (C) -1 (D) $-\frac{1}{2}$ (E) $\frac{1}{3}$

5. Find the next number in the series 2, 9, 23, 44, 72, ___.

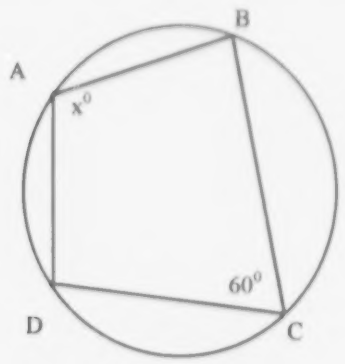
   (A) 82 (B) 107 (C) 84 (D) 83 (E) 100

6. Which of the following is the smallest?

   (A) $99\frac{9}{10}\%$ (B) $.99\overline{9}$ (C) $\frac{\frac{9}{10}}{9}$ (D) $\frac{9}{10}$ (E) $\left(\frac{9}{10}\right)^2$

7. ABCD is an inscribed quadrilateral. Find x.

   (A) 120^0 (B) 110^0 (C) 130^0 (D) 125^0 (E) 115^0



8. A number is removed from the set {1, 3, 5, 7, 9}. The remaining four numbers are such that none of them is equal to the sum of the other three, and no two of them have the same sum as the other two. Which number was removed?

(A) 9 (B) 7 (C) 5 (D) 3 (E) 1

9. Frank travels 10 miles north, 5 miles west, and finally 2 miles north again to reach his destination. How far is Frank from his starting point?

(A) 10 miles (B) 11 miles (C) 13 miles (D)17 miles (E) 19 miles

10. A wheel of radius 5 inches turns on a stationary axis at the rate of 300 rpm (revolutions per minute). Through what distance along a circular path does a fixed point on the circumference of the wheel travel in 1 second?

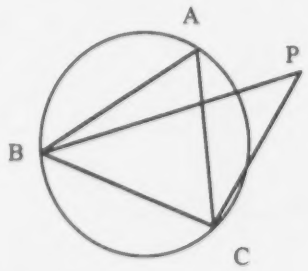
(A) 10π inches (B) 50π inches (C) 50 inches (D) 120 inches (E) 120π inches

11. If each side of a square field were five units longer, the field would contain 425 square units more surface. The number of units in the side of the original square is

(A) 40 (B) 1600 (C) 45 (D) 160 (E) 2025

12. If ΔABC is an equilateral triangle inscribed in a circle, and P is any point outside the circle, then $m\angle BPC$ is

(A) greater than 60^0 (B) less than 60^0 (C) equal to 90^0
(D) an obtuse angle (E) any of these



13. If a pipe three inches in diameter empties a tank of water in four minutes, how long would it take a pipe two inches in diameter to empty the tank?

(A) $1\frac{1}{2}$ minutes (B) $1\frac{7}{9}$ minutes (C) $2\frac{2}{3}$ minutes (D) 6 minutes (E) 9 minutes

14. The combined weight of a bottle and a tumbler equals the weight of a pitcher. The weight of the bottle equals the weight of a plate and the tumbler. Two pitchers weigh the same as three plates. How many tumblers will balance one bottle?

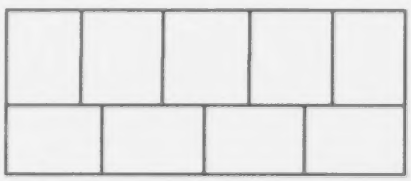
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

15. One bell chimes every 5 seconds, another every 6 seconds. If they chime simultaneously, in how many seconds will they first chime together again?

(A) 11 (B) 15 (C) 24 (D) 30 (E) 60

16. In the adjoining diagram, all 9 small rectangles have the same dimensions. The area of the large rectangle is 45. Find the perimeter of the large rectangle.

(A) 28 (B) 29 (C) 36 (D) $12\sqrt{5}$ (E) none of these



17. A cylindrical can $2\frac{1}{2}$" in diameter and 4" high holds 10 ounces of soup. How many ounces will there be in a can 3" in diameter and 5" high?

(A) 12 (B) 14 (C) 15 (D) 16 (E) 18

18. If $\frac{1}{4} + 4\left(\frac{1}{12} + \frac{1}{x}\right) = 1\frac{3}{4}$, find the numerical value of $3 + 48\left(\frac{x + 12}{12x}\right)$.

(A) $1\frac{3}{4}$ (B) $5\frac{1}{4}$ (C) 6 (D) 7 (E) 21

19. The number of new dwelling units per 10,000 people that were erected in 1956 was 25 in City A, and 35 in City B. The population of City A in 1956 was 250,000. Which of the following statements can be derived from the facts presented?

(A) More new dwelling units were built in 1956 in City B than in City A.
(B) The population of City B in 1956 was 350,000.
(C) The average age of dwelling units was reduced more in 1956 in City B than in City A.
(D) 350 new dwelling units were built in 1956 in City B.
(E) 625 new dwelling units were built in 1956 in City A.

20. Ernie can build a certain wall in 36 hours. After he has worked 20 hours, Alice joins him, and they finish the wall in 10 hours more. If they had done the job by working together the entire time, they would have finished in

(A) $22\frac{1}{2}$ hr (B) 60 hr (C) 30 hr (D) 48 hr (E) 20 hr

MATHEMATICS TEST 3
Time Limit: 30 Minutes

1. All of the following are correct except:

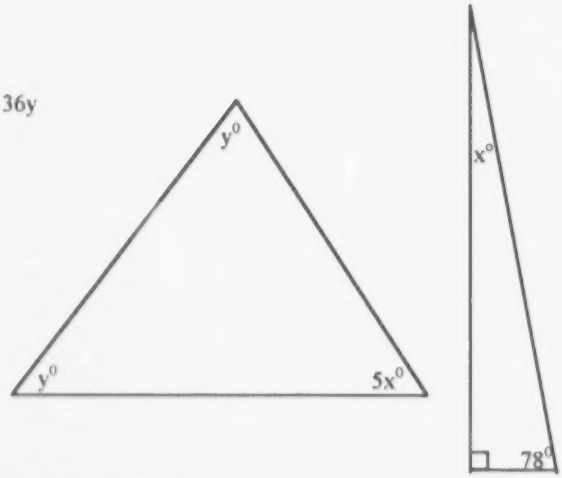
(A) $5 < 14$ (B) $-5 > -14$ (C) $\frac{1}{5} > \frac{1}{14}$ (D) $14 > 5$ (E) $-\frac{1}{5} > -\frac{1}{14}$

2. How many inches are there in y yards?

(A) $\frac{y}{36}$ (B) $\frac{y}{12}$ (C) 3y (D) 12y (E) 36y

3. In the adjoining figures, y = ?

(A) 30^0 (B) 60^0 (C) 70^0 (D) 50^0 (E) 120^0



4. If Evelyn receives a 20% cut in wages, she may gain her original pay exactly by obtaining a raise of

(A) 20% (B) 25% (C) $22\frac{1}{2}\%$ (D) \$20. (E) \$25.

5. A rectangular field is half as wide as it is long and is completely enclosed by x yards of fencing. The area in terms of x is

(A) $\frac{x^2}{2}$ (B) $2x^2$ (C) $\frac{2x^2}{9}$ (D) $\frac{x^2}{18}$ (E) $\frac{x^2}{72}$

6. Al lives $\frac{2}{5}$ of a mile from his school. If he walks to school in 72 minutes, what is his speed in miles per hour?

(A) 2 (B) $2\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $3\frac{1}{2}$ (E) $4\frac{4}{5}$

7. $\dfrac{\frac{1}{x} - \frac{1}{y}}{\frac{1}{xy}}$ = ?

(A) x - y (B) x + y (C) $\frac{x + y}{xy}$ (D) y - x (E) 2y

8. If 12 carpenters can do a job in 5 days, then 8 carpenters can do the same job in

(A) $7\frac{1}{2}$ days (B) $3\frac{1}{2}$ days (C) $2\frac{1}{2}$ days (D) 3 days (E) 9 days

9. Find the next term in the following series: 0, 2, 6, 12, 20, 30, ____, . . .

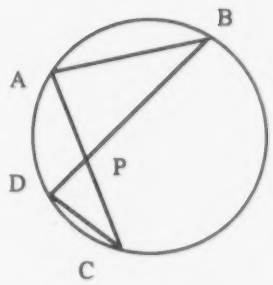
(A) 40 (B) 60 (C) 50 (D) 32 (E) 42

10. If x is 25% of y, and y is 80% of z, then x is what per cent of z?

(A) 30 (B) 20 (C) 40 (D) 45 (E) 60

11. In the adjoining diagram, $m\angle ABD = 20^0$, and $m\angle DPC = 70^0$. If points A, B, C, and D all lie on the circle, find the measure of $\angle BDC$.

(A) 30^0 (B) 100^0 (C) 50^0 (D) 120^0 (E) 90^0



12. The 25 boys in a class have an average grade of 80. The 15 girls in the same class have an average grade of 72. What is the average grade for the entire class of 40 students?

(A) 75 (B) 76 (C) 76.5 (D) 77 (E) 77.5

13. Susan bought a yard each of red, white, and blue cloth for a total of \$7.50. Later, she bought $\frac{1}{3}$ of a yard of the red, $\frac{1}{2}$ of a yard of the blue, and $\frac{1}{3}$ of a yard of the white for \$3.00. Find the price of a yard of the blue cloth.

(A) \$2.00 (B) \$2.50 (C) \$3.00 (D) \$3.50 (E) impossible to find

14. The number of ounces of water needed to reduce 9 ounces of shaving lotion containing 50% alcohol to a lotion containing 30% alcohol is

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

15. If a clerk can fill 100 envelopes an hour and the workday is 7 hours, how many clerks would be needed to fill 14,000 envelopes in 5 days?

(A) 4 (B) 20 (C) 28 (D) 100 (E) 140

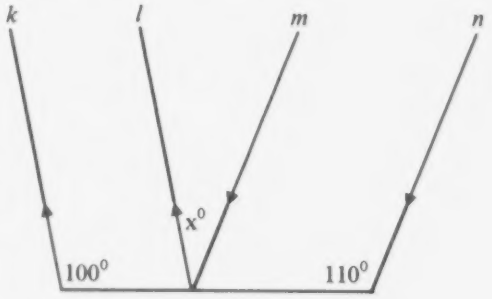
16. Al and Bob each have the same amount of wire fencing to enclose a garden. Al makes his into a square; Bob makes a rectangle with one side 3 yards longer than the side of Al's square. Which enclosure is larger, and by how much?

(A) Al's by 9 square yards (B) Bob's by 6 square yards (C) they are the same
(D) Al's, but you cannot tell by how much (E) Bob's, but you cannot tell by how much

17. A hospital which has 650 beds provides beds for people with general ailments, infectious diseases, and children's diseases. There are four times as many beds for general ailments as for infectious diseases, and 50% more beds for children's diseases than for infectious diseases. Find the number of beds for children's diseases.

(A) 150 (B) 130 (C) 100 (D) 170 (E) 200

18. In the adjoining diagram, line l is parallel to line k, and line m is parallel to line n. Find $\angle x$.

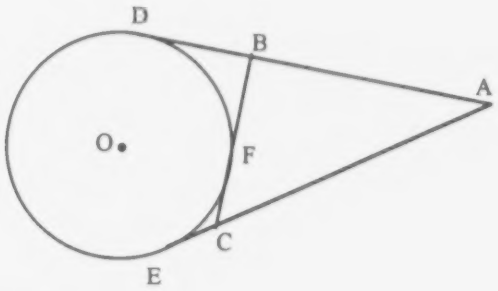


(A) 10^0 (B) 20^0 (C) 30^0 (D) 40^0 (E) 50^0

19. Two bottles are filled with two kinds of medicine mixed in the first bottle in the ratio 3:5 and in the second in the ratio 3:7. If a mixture is to be formed by taking some medicine from each bottle so that the medicine will consist of 6 ounces of the first kind and 12 ounces of the second kind, find the number of ounces taken from the first bottle.

(A) 10 (B) 9 (C) 8 (D) 7 (E) 6

20. In the diagram below, $\overline{AD}$, $\overline{AE}$, and $\overline{BC}$ are each tangent to circle O. If AD = 20, find the perimeter of triangle ABC.



(A) 20 (B) 30 (C) 40 (D) $35\frac{1}{2}$ (E) cannot be determined from the information given

MATHEMATICS TEST 4
Time Limit: 30 Minutes

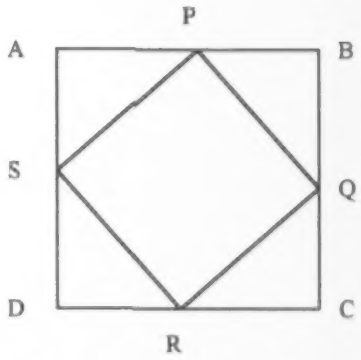
1. What percent is x of y?

(A) $\frac{x}{y}$ (B) $\frac{y}{x}$ (C) $\frac{100x}{y}$ (D) $\frac{100y}{x}$ (E) $\frac{y}{100x}$

2. If $4a - 3b = 7$ and $3a + 2b = 19$, what is the value of $14a - 2b$?

(A) 55 (B) 26 (C) 52 (D) 12 (E) none of the answers given

3. Points P, Q, R, and S are each midpoints of the sides of square ABCD. The area of ABCD is 4. What is the area of PQRS?



(A) 1 (B) 2 (C) 3 (D) $\sqrt{2}$ (E) $\sqrt{3}$

4. Max spends $\frac{1}{3}$ of his money for books, $\frac{1}{4}$ of his money for food, and $\frac{1}{6}$ of his money at the ball game. What percent of his money does he have left?

(A) $12\frac{1}{2}\%$ (B) $14\frac{2}{7}\%$ (C) $16\frac{2}{3}\%$ (D) 25% (E) $33\frac{1}{3}\%$

5. If the sum of three consecutive integers is 45, what is the sum of the next three consecutive integers?

(A) 48 (B) 46 (C) 51 (D) 54 (E) 57

6. If two concentric circles have diameters of 10 and 12, the area between the two circles is

(A) 44π (B) 2π (C) 11π (D) π (E) 4π

7. How many 6-ounce glasses of juice can be poured from 12 quarts of juice? (1 quart = 32 ounces)

(A) 2 (B) 16 (C) 36 (D) 64 (E) 75

8. The angle formed by the hour and minute hands of a clock at 2:15 is

(A) 30^0 (B) $27\frac{1^0}{2}$ (C) $157\frac{1^0}{2}$ (D) $172\frac{1^0}{2}$ (E) $22\frac{1^0}{2}$

9. If each dimension of a rectangle is increased by 10%, the area is increased by

(A) 10% (B) 20% (C) 21 % (D) 110% (E) 121 %

10. If a chord of a circle of radius 5 inches is 3 inches from the circle's center, then the length of the chord in inches is

(A) 8 (B) 4 (C) 6 (D) 2 (E) cannot be determined

11. A teacher is forming committees in her class of 30 children. The committees may be composed of 4, 5, 7, or 8 children. What is the least number of committees that the teacher can form so that every child will be on at least one committee?

(A) 3 (B) 4 (C) 6 (D) 7 (E) 8

12. If the sum of three consecutive odd integers is 381, then one of these integers is

(A) 117 (B) 119 (C) 121 (D) 123 (E) 125

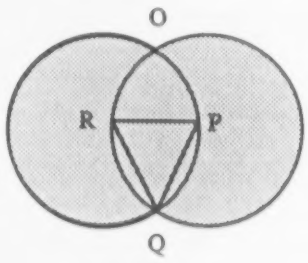
13. On a map whose scale is $1\frac{1}{2}$ inches = 400 miles, there is a circle whose diameter is $\frac{3}{16}$ inch. The answer listed below which gives the closest approximation to the number of square miles of land represented by the circular map area is

(A) 67,500 (B) 196 (C) 1,960 (D) 3,920 (E) 7,000

14. Five stones are placed in a straight line on the ground at intervals of 5 feet. A basket is located 10 feet from the end stone on the extension of the line of stones. If a runner starts from the basket, picks up the stones, and carries them one at a time to the basket, the total distance in feet the runner must have covered in order to place all of the stones in the basket is

(A) 60 (B) 100 (C) 120 (D) 200 (E) none of these

15. In the adjoining diagram, the circles are centered at points P and R, respectively, and intersect at points O and Q. If side $\overline{PQ}$ of equilateral triangle ΔPQR has length 3, find the perimeter of the shaded region.



(A) $\frac{9\pi}{2}$ (B) 9π (C) 8π (D) 12π (E) 16π

16. The reading of a certain instrument is the average of 6 successive positions of the indicator. If variations on either side of 24 are +0.3, -0.7, +0.2, -0.5, +0.2, and -0.4, what is the reading of the instrument?

(A) -9 (B) 23.1 (C) 23.85 (D) 24.15 (E) 24.9

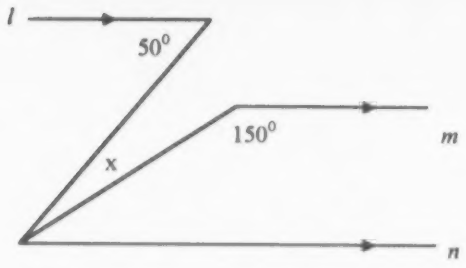
17. When the circumference of a circle is increased from 20 inches to 25 inches, the radius is increased by

(A) 5 inches (B) $2\frac{1}{2}$ inches (C) $\frac{5}{\pi}$ inches (D) $\frac{5}{2\pi}$ inches (E) $\frac{\pi}{5}$ inches

18. In the adjoining diagram, the ratio of the diameters of the three semicircles is 2:3:5. Find the ratio of the area of the shaded region to the area of the largest semicircle.

(A) 3:2 (B) 12:25 (C) 13:25 (D) 5:2 (E) 12:13

19. In the adjoining diagram, if lines l, m, and n are parallel, then $m\angle x =$



(A) 50^0 (B) 30^0 (C) 20^0 (D) 60^0
(E) cannot be determined

20. A camera is left open for 2 hours at night. During this time a star makes a photographic trail $\frac{1}{2}$ inch long on the film. This trail would form a complete circle after 24 hours. What is the radius of this circle in inches?

(A) $\frac{6}{\pi}$ (B) $\frac{3}{\pi}$ (C) 3π (D) 6π (E) none of these

MATHEMATICS TEST 5
Time Limit: 30 Minutes

1. Which of the following results in a positive number?

(A) (+7) + (-8)
(B) (-8) - (+4)
(C) (-3) - (-7)
(D) (-2) • (-2) • (-5)
(E) (+5) + (+7) - (+13)

2. Which of the following is the sequence of remainders when 7, 14, 21, 28, and 35 are divided by 3?

(A) 1, 0, 2, 0, 1
(B) 1, 1, 1, 1, 1
(C) 4, 11, 18, 25, 32
(D) 1, 2, 3, 4, 5
(E) 1, 2, 0, 1, 2

3. Ruth can produce n instruments in h hours. How many instruments can she produce in 1 hour?

(A) $\frac{h}{n}$ (B) $\frac{n}{h}$ (C) nh (D) n + h (E) $\frac{n+h}{2}$

4. Which of the following represents the average of all the integers from 20 to 29, inclusive?

(A) 27 (B) $26\frac{1}{2}$ (C) 25 (D) 24 (E) $24\frac{1}{2}$

5. The difference between one half of a number and one seventh of the same number is 5. What is the number?

(A) 10 (B) 70 (C) 12 (D) 14 (E) none of these

6. The circumference of a circle is increased by 20%. Find the percent of increase in the area of the circle.

(A) 40% (B) 40π% (C) 20% (D) 36% (E) 44%

7. If b = $83\frac{1}{3}$% of c, and a = 60% of b, what percent is a of c?

(A) $\frac{1}{2}$ (B) $52\frac{1}{2}$ (C) $55\frac{5}{9}$ (D) $58\frac{1}{3}$ (E) 50

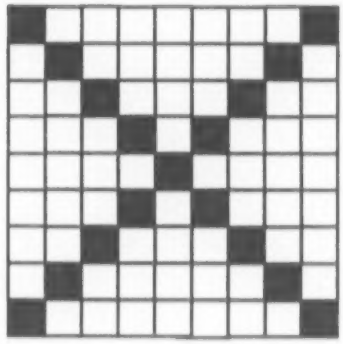
8. Ten gallons of gasoline were poured into a tank already $\frac{2}{3}$ full. It was then found to be $\frac{5}{6}$ full. How many gallons of gasoline does the tank hold?

(A) 12 (B) 15 (C) 18 (D) 36 (E) 60

9. Fred has 5 hours at his disposal. How far (in miles) can he ride his bicycle, going at 10 miles per hour, and returning by the same route at 4 miles per hour?

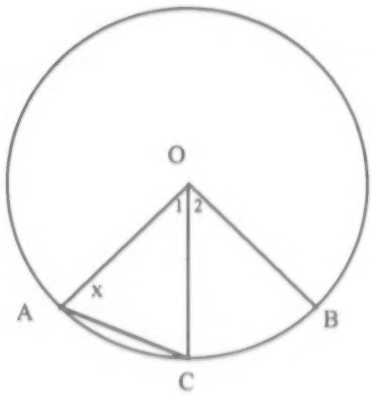
(A) $9\frac{3}{5}$ (B) $10\frac{1}{2}$ (C) $12\frac{3}{4}$ (D) $14\frac{2}{7}$ (E) $15\frac{1}{7}$

10. Which of the following best represents the percent of the adjoining figure which is shaded? (Assume that all the squares are equal in area.)



(A) 15% (B) 21% (C) 25% (D) 31% (E) 79%

11. In the circle at the right with O as center, $\angle AOB$ is a right angle. If $m\angle 1 = m\angle 2$, find $m\angle x$. (Note: A, B, and C lie on the circle.)



(A) $22\frac{1^0}{2}$ (B) 45^0 (C) $67\frac{1^0}{2}$ (D) 135^0
(E) cannot be determined with information given

12. The sum of six numbers is 2,367. Three of these six numbers are: 123, 456, and 789. What is the average of the remaining three numbers

(A) 394.5 (B) 459 (C) 396 (D) 333
(E) cannot be determined with information given

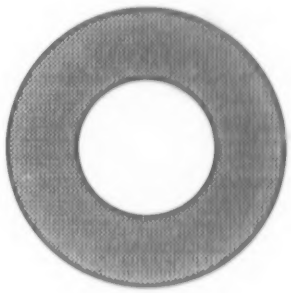
13. If there are 6 people in a room, and each person shakes hands with everyone else exactly once, how many handshakes are there?

(A) 6 (B) 15 (C) 30 (D) 12 (E) 18

14. The length of a rectangular piece of tin is twice its width. A two-inch square is cut from each corner so that a box may be formed by folding up the sides. If the box has a volume of 60 cubic inches, what was the width of the original piece of tin?

(A) 7" (B) 6" (C) 10" (D) 14" (E) 15"

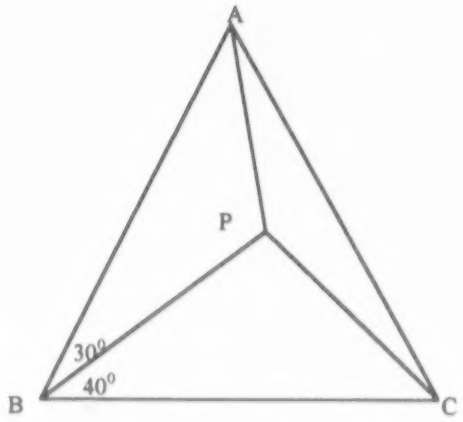
15. In the adjoining diagram, the width of the shaded region is exactly equal to the radius of the smaller circle. (The circles are concentric.) What part of the area of the larger circle is the area of the shaded region?



(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) $\frac{3}{4}$

16. Given triangle ABC. Point P, in the plane of the triangle, is equidistant from the vertices of the triangle. If $m\angle ABP = 30^0$, and $m\angle CBP = 40^0$, which of the following represents the number of degrees in the measure of $\angle PAC$?

(A) 20^0 (B) 30^0 (C) 40^0 (D) 22^0
(E) cannot be determined from information given



17. Giselle buys 3 dollars worth of each of two different kinds of pencils, priced at 4 and 6 cents per pencil, respectively. The average price in cents paid for a pencil is

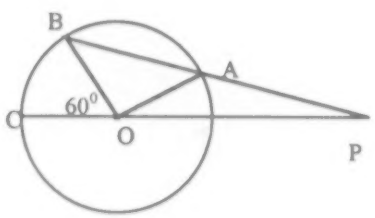
(A) 4.5 (B) 4.8 (C) 5 (D) 5.1 (E) 5.2

18. Herb walks half the distance to school at a rate of 4 feet per second, and then doubles his rate for the rest of the way so that he won't be late. Find his average rate in feet per second for the whole distance.

(A) 5 (B) $5\frac{1}{3}$ (C) $5\frac{1}{2}$ (D) 6 (E) 15

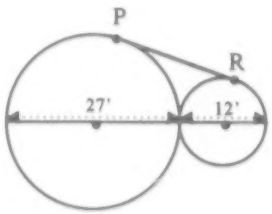
19. In the adjoining diagram, $\overline{AO}$, $\overline{BO}$, and $\overline{CO}$ are radii of circle O. AP = the radius, and POC is a straight line. If $m\angle BOC = 60^0$, what is the measure, in degrees, of $\angle P$?

(A) 10^0 (B) 15^0 (C) 20^0 (D) 30^0 (E) 40^0



20. Two circles of diameter 12 ft and 27 ft are externally tangent. Find the length of one of their common external tangents.

(A) 9 ft (B) 18 ft (C) 6 ft (D) 15 ft (E) $14\frac{1}{2}$ ft



MATHEMATICS TEST 6
Time Limit: 30 Minutes

1. If $x < 1 < y$, then which of the following is not necessarily true?
(A) $-x > -1$ (B) $x < y$ (C) $0 < x$ (D) $-1 < y$ (E) $1 > x$

2. If the difference of two nonzero numbers is 0, then their quotient is

(A) 0 (B) 1 (C) -1 (D) any real number (E) cannot be determined with information given

3. How many fifths are in $5\frac{7}{10}$?

(A) $26\frac{2}{5}$ (B) $\frac{57}{50}$ (C) $28\frac{1}{2}$ (D) 29 (E) 39

4. If pens cost p cents each, how many can be bought for a cents?

(A) $\frac{a}{p}$ (B) $\frac{p}{a}$ (C) $a + p$ (D) ap (E) none of these

5. $\frac{1}{60}$ of the units produced in a factory are spoiled. By introducing a new machine, $\frac{1}{5}$ of the spoilage could be prevented. What fraction of the units produced in a factory are spoiled.

(A) $\frac{1}{75}$ (B) $\frac{1}{300}$ (C) $\frac{4}{5}$ (D) $\frac{1}{12}$ (E) none of these

6. The average of 3 fractions is $\frac{1}{4}$. Two of them are $\frac{1}{3}$ and $\frac{1}{4}$. What is the other fraction?

(A) $1\frac{1}{3}$ (B) $\frac{1}{5}$ (C) $\frac{1}{6}$ (D) $\frac{3}{14}$ (E) none of these

7. If $\frac{2a}{b} = 7$, find the value of $\frac{2a - b}{b}$.

(A) 6 (B) 5 (C) $4\frac{3}{4}$ (D) 7 (E) impossible to find

8. What percent of 2 hours is 5 seconds?

(A) $\frac{5}{36}\%$ (B) $\frac{5}{7200}\%$ (C) $\frac{5}{72}\%$ (D) $\frac{5}{3600}\%$ (E) none of these

9. If d is the length of a diagonal of a rectangle and s is the length of one side, express the area of the rectangle in terms of d and s.

(A) $s\sqrt{d^2 - s^2}$ (B) ds (C) $s^2\sqrt{d^2 - s^2}$ (D) $\frac{1}{2}d^2$ (E) none of these

10. A square piece of cloth with side of length 16" is folded so that each corner meets at the center of the square. What are the dimensions of the new figure?

(A) $8\sqrt{2}$ " sides (B) 8" sides (C) $16\sqrt{2}$ " sides (D) 4" sides (E) $4\sqrt{2}$ " sides

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11. The pages of a certain book are numbered consecutively, and there are no blank pages. Noreen, reading this book, begins on the top of page P and ends her reading on the bottom of page Q. How many pages has Noreen read?

(A) P + Q (B) Q - P (C) Q - P + 1 (D) P - Q + 1 (E) cannot be determined from information given

12. Consider the ordered set S: S = (5, 6, 7, 8, x)
If x is equal to the average of the three center elements, what is the average of all the elements?

(A) 5 (B) 5.6 (C) 6 (D) 6.6 (E) 7

13. A rectangular photograph whose dimensions are $1\frac{7''}{8}$ by $2\frac{1''}{2}$ is to be enlarged so that the larger dimension will he 4". How long will the shorter dimension become?

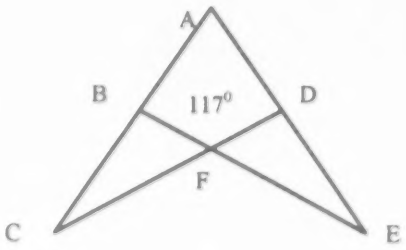
(A) $2\frac{1''}{2}$ (B) $2\frac{3''}{8}$ (C) 3" (D) $3\frac{3''}{8}$ (E) none of these

14. Which of the following is the smallest fraction that each of $\frac{6}{5}$, $\frac{3}{10}$, and $\frac{4}{15}$ will divide exactly?

(A) $\frac{72}{5}$ (B) $\frac{72}{750}$ (C) $\frac{12}{5}$ (D) $\frac{12}{15}$ (E) $\frac{12}{30}$

15. In the adjoining diagram, AC = CD and BE = AE. Points A, B, and C are collinear, and points A, D, and E are collinear. $m\angle BFD = 117^0$. Find the measure, in degrees, of $\angle A$.

(A) 73^0 (B) 81^0 (C) 90^0 (D) 117^0 (E) none of these



16. Points A, B, and C are on circle O, such that A is the center of arc BC, B is the center of arc AC and C is the center of arc BA. The shortest distance from A to B is 4 inches. Find the area of ΔABC in square inches.

(A) 4 (B) $4\sqrt{3}$ (C) 8 (D) $8\sqrt{3}$ (E) none of these

17. Find the value (in simplest form) of: $\dfrac{1}{1 + \dfrac{1}{2 + \dfrac{1}{3}}}$

(A) $\frac{5}{6}$ (B) $\frac{6}{11}$ (C) $\frac{11}{6}$ (D) $\frac{7}{10}$ (E) $\frac{10}{7}$

18. You have a nickel, a dime, a quarter, and a fifty-cent piece. A clerk shows you several articles, each of which has a different price, any one of which you could buy with coins without receiving change. What is the largest number of articles the clerk could have shown you?

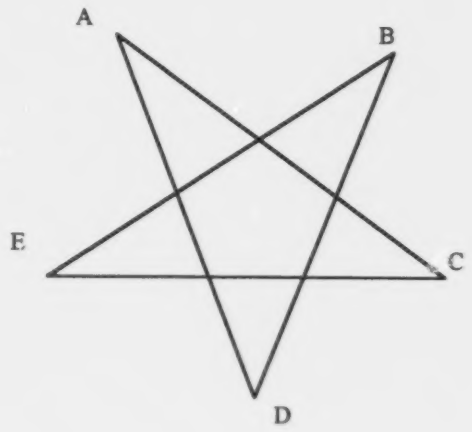
(A) 8 (B) 10 (C) 13 (D) 15 (E) 21

19. Together, X, Y, and Z can do a piece of work in 8 hours. X and Y can do it in 12 hours, and X does as much work as Y and Z together. How much time does each require to do the entire job alone?

(A)	(B)	(C)	(D)	(E)
X in 5 hr	X in 24 hr	X in 16 hr	X in 36 hr	X in 24 hr
Y in 7 hr	Y in 16 hr	Y in 48 hr	Y in 48 hr	Y in 16 hr
Z in 17 hr	Z in 8 hr	Z in 24 hr	Z in 24 hr	Z in 40 hr

20. In the plane figure at the right, which of the following represents the sum of the measures of the interior angles at vertices A, B, C, D, and E?

(A) 180^0 (B) 360^0 (C) 540^0 (D) 720^0
(E) cannot be determined from information given



MATHEMATICS TEST 7
Time Limit: 30 Minutes

1. $14\frac{1}{8} - .25 = ?$

(A) 13.875 (B) $14\frac{3}{8}$ (C) 14.1 (D) 13.85 (E) 14

2. $7 \times 3 \times \frac{1}{5} = \frac{4}{?}$

(A) $1\frac{1}{20}$ (B) $\frac{84}{5}$ (C) $\frac{21}{5}$ (D) $\frac{4}{105}$ (E) $\frac{20}{21}$

3. Which of the following is equal to ab + cd + bd + ac?

(A) 2abc + 2bcd (B) a(b + c + cd) (C) ad + bc (D) (a + d)(b + c) (E) 2ab + 2cd

4. If $\frac{1}{800}\,t = \frac{1}{24}$, find the value of t.

(A) 824 (B) $33\frac{1}{3}$ (C) 776 (D) 35 (E) $3\frac{1}{3}$

5. A swimming pool is W feet wide and L feet long. How many feet deep is the pool if it contains C cubic feet of water?

(A) $\frac{LW}{C}$ (B) $\frac{C}{LW}$ (C) LWC (D) $\frac{L\sqrt[3]{C}}{W}$ (E) $\sqrt[3]{C}$

6. Kenny, walking to school daily, finds that he takes 352 steps per minute. He measures each step to be $1\frac{1}{2}$ feet long. What is Kenny's speed in miles per hour?

(A) 5 (B) $5\frac{1}{2}$ (C) 6 (D) $6\frac{1}{2}$ (E) cannot be determined with information given

7. If $x\sqrt{.04} = 1$, then x equals

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{1}{5}$ (D) 5 (E) .05

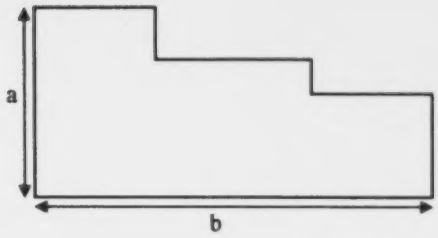
8. If $x^2 + 2xy + y^2 = 2$, then x + y equals

(A) ± 2 (B) $\pm \sqrt{2}$ (C) 0 (D) $\frac{1}{2}$ (E) 4

9. The area of $\Delta XYZ = 60$ square inches. $m\angle Y = 90^0$. YZ = 8". Find XZ.

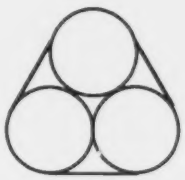
(A) $\sqrt{120\frac{1}{4}}$" (B) 13" (C) 15" (D) 17" (E) $\sqrt{113\frac{1}{4}}$"

10. What is the perimeter of the adjoining figure? (All lines are either perpendicular or parallel.)



(A) ab (B) a + 3b (C) $a^2 + b^2$ (D) 2(a + b)
(E) a + b

11. Three round barrels, each with a radius of $1\frac{1}{2}$ ft, are tied together as shown in the diagram. How long a rope is needed to go around them exactly once?



(A) $9 + 2\frac{1}{4}\pi$ ft (B) 12π ft (C) $3\pi + 9$ ft (D) $12\sqrt{3}$ ft
(E) $12\pi\sqrt{3}$ ft

12. A truck can carry between $3\frac{1}{2}$ and $4\frac{1}{2}$ tons. What is the largest number of crates weighing 100 lb each that it will carry? (1 ton = 2000 lb)

(A) 900 (B) 45 (C) 70 (D) 80 (E) 90

13. Which of the following operations will always leave a triangle whose area is the same as the original one?

I. Change the length of two sides but keep the lengths of the base and height the same.
II. Double the length of the height and take one half of the length of the base
III. Double the length of the base and take one half of the length of the height

(A) none (B) I only (C) II and III only (D) III only (E) I, II, and III

14. What is the least number of dots which may be moved to make the adjoining diagram reverse its direction (i.e., rotate 180^0)?

(A) 3 (B) 4 (C) 5 (D) 6 (E) 8

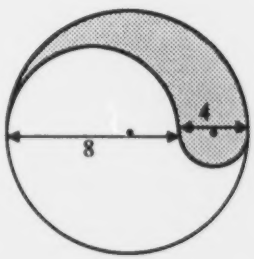
15. Donald sold two books for \$6.00 each. On one book he gained 20% whereas on the other he lost 20%. How much did he gain or lose in this transaction?

(A) gained \$.50 (B) lost \$.50 (C) gained \$1.20 (D) lost \$1.20
(E) did not gain or lose, but was even

16. The areas of the front, the side, and the bottom of a rectangular box are known. If P is the product of these three areas, then the volume of the box, in proper units, equals

(A) $\sqrt{P}$ (B) $\sqrt[3]{P}$ (C) $\frac{P}{3}$ (D) P (E) none of these

17. In the adjoining diagram, find the area of the shaded region formed by semicircles with diameters 8 and 4, respectively.



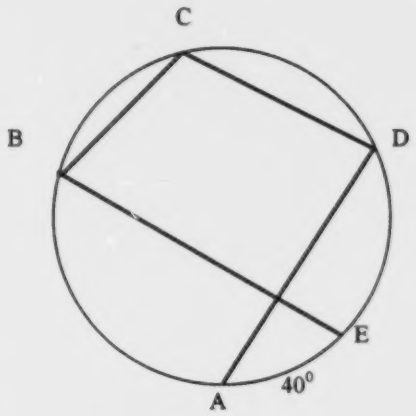
(A) 6π (B) 10π (C) 12π (D) 18π (E) 24π

18. A train, traveling at the uniform rate of 30 mph, enters a tunnel twice the length of the train. Six minutes elapse from the moment the first car enters the tunnel to the moment the last car clears the tunnel. The length of the train in miles is

(A) 2 (B) $1\frac{1}{2}$ (C) 1 (D) $\frac{3}{4}$ (E) $\frac{1}{2}$

19. In the adjoining diagram, if the measure of arc AE = 40^0, then $m\angle B + m\angle D = ?$

(A) 320^0 (B) 160^0 (C) 150^0 (D) 200^0 (E) 250^0



20. A hungry hunter came upon two shepherds, one of whom had 3 loaves of bread and the other 5, all of the same size. The loaves were divided equally among the three, and the hunter paid 8 cents for his share. How should the shepherds divide the money?

(A) 3 cents and 5 cents (B) 2 cents and 6 cents (C) 1 cent and 7 cents
(D) $1\frac{1}{2}$ cents and $6\frac{1}{2}$ cents (E) none of the above

MATHEMATICS TEST 8
Time Limit: 30 Minutes

1. If p = q, find the value of 7 + 3(p - q).

(A) 7 (B) 10 (C) 7 + 6p (D) 7 - 6q (E) 0

2. 56 is $\frac{7}{8}$ of which of the following numbers?

(A) 63 (B) 64 (C) 49 (D) 1 (E) $\frac{8}{7}$

3. If -431x -1250x + 436x + 995x = 500, then x = ?

(A) 2 (B) $\frac{1}{2}$ (C) -2 (D) $-\frac{1}{2}$ (E) $\frac{500}{351}$

4. Emilie saved $2.50 in buying a pair of jeans on sale. If she spent $25 for the jeans, she saved about:

(A) 8% (B) 9% (C) 10% (D) 11% (E) 12%

5. One term does not belong in the following series: 1, 8, 3, 6, 9, 4, 7, 2. Which of the following numbers should replace the incorrect term?

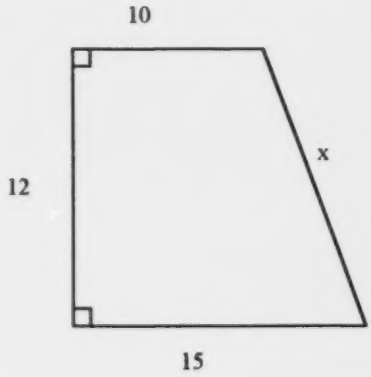
(A) 9 (B) 6 (C) 2 (D) 5 (E) 0

6. Evaluate the following: (211 X 555) + (445 X 789) + (555 X 789) + (211 X 445)

(A) 1,424,578 (B) 2,000,000 (C) 555,000 (D) 100,000 (E) 1,000,000

7. Find the value of x in the adjoining figure.

(A) 10 (B) 11 (C) 12 (D) 13 (E) 14



8. A 5-lb box of sugar used to cost 45¢. A 2-lb box now costs 25¢. Find the ratio of the present price per pound to the former one.

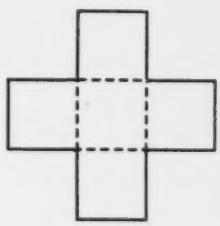
(A) 9:12.5 (B) 12:9 (C) 9:5 (D) 25:18 (E) none of these

9. After sailing 400 miles directly southeast, a ship is how many miles south of its starting point?

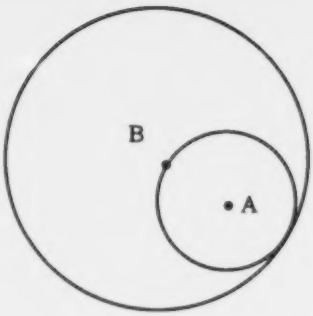
(A) 200 (B) $200\sqrt{2}$ (C) $200\sqrt{3}$ (D) $300\sqrt{2}$ (E) $400\sqrt{2}$

10. The total area of the adjoining figure, which is composed of five equal squares, is 80. Find the perimeter of the figure.

(A) 12 (B) 24 (C) 36 (D) 48 (E) none of these



11. Circle A is internally tangent to circle B and also passes through its center. What is the ratio of the circumference of circle B to the circumference of circle A?



(A) $\frac{1}{4}$ (B) $\frac{4}{1}$ (C) $\frac{\pi}{2}$ (D) $\frac{2}{1}$ (E) $\frac{1}{2}$

12. During which of the following time intervals will the minute hand pass over the hour hand?

(A) 5:00 - 5:26 (B) 5:26 - 5:27 (C) 5:27 - 5:28 (D) 5:28 - 5:29
(E) 5:29 - 6:00

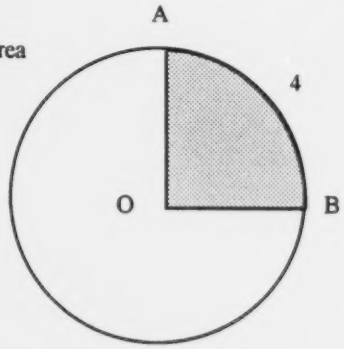
13. The base of a triangle is twice as long as a side of a square, and the areas of the two figures are the same. The ratio of the length of the altitude of the triangle to the length of the side of the square is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) 4

14. A clock sounds 6 o'clock by striking 6 times in 5 seconds. How many seconds does the clock take to strike 12 times at noon? Assume each striking consumes no time.

(A) 10 seconds (B) 11 seconds (C) 12 seconds (D) 13 seconds (E) 9 seconds

15. In circle O with radius of $\frac{8}{\pi}$, what part of the area of the circle is the area of the sector? (length of arc AB = 4)

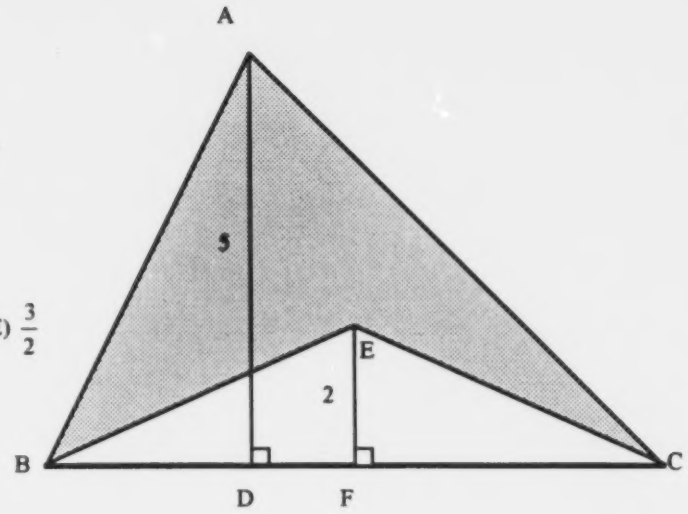


(A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{1}{6}$ (D) $\frac{3}{4}$ (E) $\frac{1}{2}$

16. A company sells 60% of its stock for cash, 25% to charge account customers, and 15% as COD sales. If the COD sales are $4000 less than the charge account sales, then the cash sales are

(A) $60,000 (B) $12,000 (C) $16,000 (D) $20,000 (E) $24,000

17. In the adjoining diagram, what is the ratio of the area of ΔABC to the area of the shaded region? AD = 5; FE = 2; $\overline{AD}$ and $\overline{FE}$ are altitudes.



(A) $\frac{5}{2}$ (B) $\frac{25}{4}$ (C) $\frac{25}{9}$ (D) $\frac{5}{3}$ (E) $\frac{3}{2}$

18. The front wheels of a wagon are 2π feet in circumference, and the back wheels are 3π feet in circumference. If it is known that in traveling a distance d, a front wheel has made 10 more revolutions than a back wheel, then the value of d in feet is

(A) 10π (B) 20π (C) 60π (D) 20 (E) impossible to determine

19. The product of a set of n numbers is P. If each member in the set is multiplied by 5, the product of the set of numbers thus obtained is

(A) 5n (B) 5P (C) 5^n (D) 5nP (E) 5^n P

20. On a 26-question test, 6 points are deducted for each wrong answer, and x points are credited for each correct answer, x being an integer less than 20. If all questions are answered, how many are correct if the score is zero?

(A) 6 (B) 9 (C) 12 (D) 15 (E) 18

MATHEMATICS TEST 9
Time Limit: 30 Minutes

1. What is the approximate value of 39,123,678 X 7,251.889712?

(A) 273,000,000 (B) 273,000 (C) 273,000,000,000 (D) 2,808,000,000 (E) 2,808,000

2. Which of the following has the greatest value?

(A) $\frac{1}{3}$ (B) $\frac{2}{9}$ (C) $\frac{1}{4}$ (D) $\frac{1}{5}$ (E) $\frac{4}{15}$

3. $(a + b)^2 = a^2 + b^2$ This statement is true:

(A) always (B) sometimes (C) never (D) only with fractions (E) cannot be determined

4. If 9 carpenters can build a house in 36 days, how many days will it take 27 carpenters to build the same house?

(A) $6\frac{3}{4}$ (B) 12 (C) 18 (D) 108 (E) 81

5. When dish towels are hung to dry on a straight line, each towel must have a clothespin at each end. However, the same pin can hold two towels when they hang next to each other. How many clothespins are then needed to hang up a dozen towels?

(A) 13 (B) 12 (C) 11 (D) 23 (E) 24

6. How many circles 1 inch in diameter will have the same total area as one circle 2 inches in diameter?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

7. When molded into a cube, 64 cubic inches of aluminum will take up how many square inches of floor space?

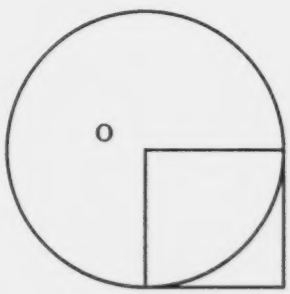
(A) 4 (B) 8 (C) 12 (D) 16 (E) 96

8. Given a set of elements, S, obeying these two laws:
I. $(a + b) + (c + d) = (a + c) + (b + d)$
II. $a + o = a = o + a$

Which of the following laws must S obey?
A. $a + b = b + a$
B. $a + (b + c) = (a + b) + c$

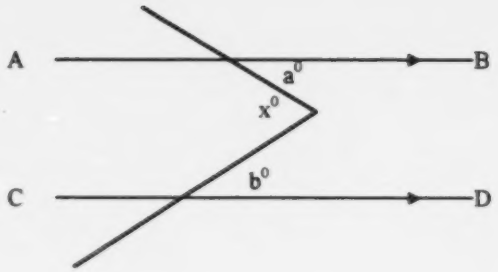
(A) only A (B) only B (C) both A and B (D) neither A nor B
(E) cannot be determined from information given

9. If K represents the area of the square in the diagram, which of the following represents the area of the circle? (Vertex O of the square is at the center of the circle.)



(A) $\pi\sqrt{K}$ (B) πK (C) πK^2 (D) $4K$
(E) cannot be determined from information given

10. In the adjoining diagram, lines AB and CD are parallel. Express the measure of $\angle x$ in terms of a and b.

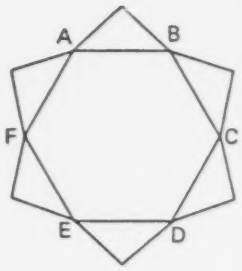


(A) $x = 180 - ab$ (B) $x = 2(a + b)$ (C) $x = b - a$
(D) $x = a - b$ (E) $x = a + b$

11. If $t^2 = 3$, then $(3t - 1)^2 = ?$

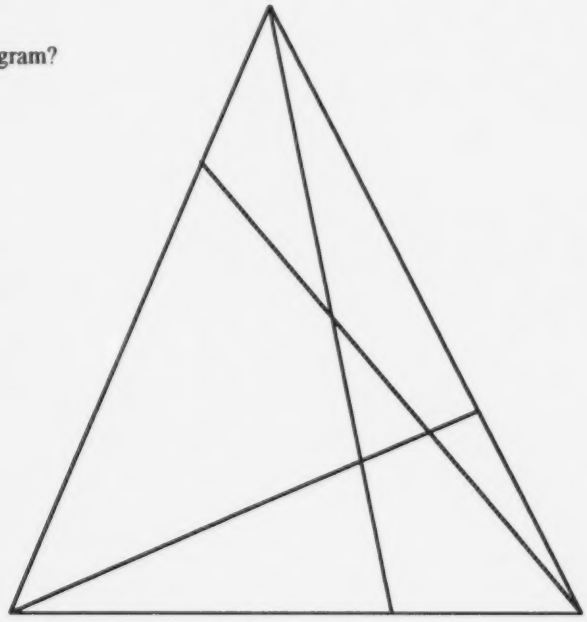
(A) $10 \pm 6\sqrt{3}$ (B) $28 \pm 6\sqrt{3}$ (C) $1 + 3\sqrt{3}$ (D) $3\sqrt{3} - 1$ (E) 8

12. On each side of regular hexagon ABCDEF, an isosceles right triangle is drawn. If the perimeter of the hexagon is 36, what is the area of the whole figure in the adjoining diagram?



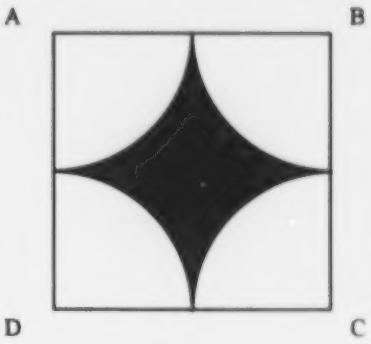
(A) 54 (B) $54\sqrt{3}$ (C) 108 (D) $108\sqrt{3} + 108$ (E) $54 + 54\sqrt{3}$

13. How many different triangles are in the adjoining diagram?



(A) 8 (B) 12 (C) 15 (D) 17 (E) 19

14. In the adjoining diagram, ABCD is a square in which congruent arcs are drawn at each vertex. The diameter of the circle from which each of these arcs was taken is congruent to a side of the square. AB = 2s. Find the area of the shaded region.



(A) $s^2(4 - \pi)$ (B) $s^2(1 - \pi)$ (C) $4\pi s^2 - 2s^2$ (D) $s^2(2 - 4\pi)$ (E) $s(2 - \pi)$

15. The four-digit number a47b, in which a and b are unknown digits, is divisible by nine. Then a + b can equal

(A) 3 (B) 4 (C) 7 (D) 8 (E) 17

16. Al owes Harvey 40¢; Harvey owes Al 50¢; Al gives Harvey 60¢. What is now needed to cancel the debt?

(A) Harvey must give Al 50¢ (B) Al must give Harvey 50¢ (C) Harvey must give Al 70¢
(D) Al must give Harvey 70¢ (E) Al must give Harvey 30¢

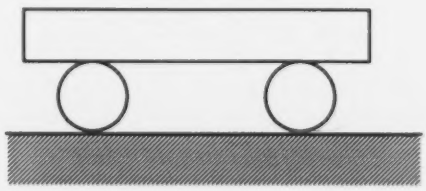
17. $\frac{1}{20}$ of the boys in a school plan to be doctors; $\frac{3}{4}$ of all the students who plan to be doctors are boys; $\frac{3}{5}$ of the students are boys. What fraction of the girls plan to be doctors?

(A) $\frac{3}{100}$ (B) $\frac{1}{40}$ (C) $\frac{1}{100}$ (D) $\frac{9}{400}$ (E) none of these

18. An army had 5 soldiers to every 6 soldiers of its opponent. During the battle, the first lost 14,000 soldiers and the second lost 6,000 soldiers. The ratio was then 2 to 3. How many did each army have after the battle?

(A) 26,000 and 39,000 (B) 36,000 and 54,000 (C) 14,000 and 21,000 (D) 20,000 and 30,000
(E) none of these

19. A slab rests on rollers, each 1 foot in diameter. How far forward does the slab move when the rollers have made one revolution? (Assume there is no slippage.)



(A) 1 foot (B) π feet (C) 2π feet (D) 3π feet
(E) 0 feet

20. Al takes 2 hours longer to travel 12 miles than Noreen, but if Al doubled his pace, he would walk it in 1 hour less than Noreen. Find Al's rate of walking in miles per hour.

(A) 10 (B) 8 (C) 6 (D) 4 (E) 2

MATHEMATICS TEST 10
Time Limit: 30 Minutes

1. How many integers greater than 0 and less than 100 are divisible by each of 2, 3, and 5?

   (A) 2 (B) 3 (C) 4 (D) 5 (E) 6

2. Arrange the following in order of size, beginning with the largest: $\frac{1}{3}, \frac{3}{8}, \frac{4}{11}$.

   (A) $\frac{1}{3}, \frac{3}{8}, \frac{4}{11}$ (B) $\frac{3}{8}, \frac{1}{3}, \frac{4}{11}$ (C) $\frac{4}{11}, \frac{3}{8}, \frac{1}{3}$ (D) $\frac{3}{8}, \frac{4}{11}, \frac{1}{3}$ (E) $\frac{4}{11}, \frac{1}{3}, \frac{3}{8}$

3. The next number in the following series is: 2, 5, 9, 14, 20, ___, . . .

   (A) 24 (B) 25 (C) 26 (D) 27 (E) 28

4. Which of the following values for x will satisfy the following equation: $\frac{x^2 - 9}{x - 3} = 0$?

   (A) $\frac{1}{2}$ (B) 1 (C) 2 (D) -3 (E) 3

5. Reduce $\frac{1 - a^2}{a - 1}$ to lowest terms.

   (A) 1 + a (B) -(a - 1) (C) -1 - a (D) a - 1 (E) none of these

6. The sum of any real negative number and its reciprocal is never greater than

   (A) -6 (B) -5 (C) -4 (D) -3 (E) -2

7. In a group of 87 people, 58 have brown hair, 63 have brown eyes, and 49 have both. How many have neither?

   (A) 34 (B) 2 (C) 9 (D) 14 (E) 15

8. If x! represents the product of all positive integers less than or equal to x, then 4! equals

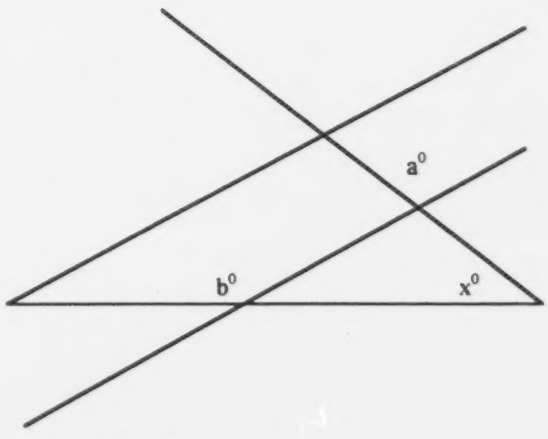
   (A) 4 (B) 6 (C) 12 (D) 24 (E) 16

9. If lighthouse A is 24 miles southeast of lighthouse B and lighthouse C is 10 miles southwest of lighthouse B, which of the following distances may represent the distance from lighthouse A to lighthouse C?

   (A) 26 (B) 34 (C) 14 (D) 37 (E) none of these

10. In the adjoining diagram, the rays of $\angle x$ are cut by two parallel lines. The measure of $\angle x$ in terms of a and b is

    (A) a - b (B) b - a (C) 180 - a + b
    (D) 180 - a - b (E) 180 - b + a

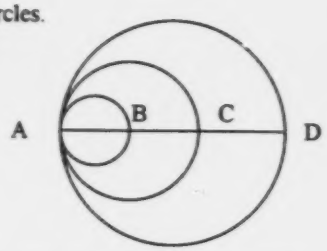


11. The number of diagonals, d, in a polygon of n sides is given by the formula: $d = \frac{n^2 - 3n}{2}$. If a polygon has 90 diagonals, how many sides does it have?

(A) 8 (B) 12 (C) 15 (D) 16 (E) 17

12. In the adjoining diagram, $\overline{AB}$, $\overline{AC}$, and $\overline{AD}$ are diameters of the three circles. If AB = BC = CD, what part of the area of the largest circle is the area of the smallest circle?

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{6}$ (D) $\frac{1}{9}$ (E) $\frac{2}{3}$



13. Lines are drawn across a rectangle parallel to the sides, as in the adjoining diagram. How many rectangles can be counted?

(A) 6 (B) 7 (C) 10 (D) 11 (E) 18

14. Consider a circle whose diameter is increased by π units. By how many units has the circumference been increased as a result of this change in the diameter?

(A) π^2 (B) π (C) $2\pi^2$ (D) 2π (E) $\frac{\pi}{2}$

15. An open box is made from a square piece of tin by cutting out a 6-inch square from each corner and turning up the sides. The box contains 150 cubic inches. Find the side of the original square.

(A) 7 inches (B) 17 inches (C) 25 inches (D) 50 inches (E) none of these

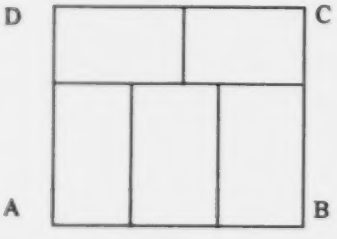
16. Find the difference between the maximum number of school days in a 31-day month and the minimum number of school days in a 30-day month with no holidays. (Assume a 5-day school week.)

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

17. In numbering the pages of a book, 3,357 digits are required. The number of pages in the book is

(A) 998 (B) 3,357 (C) 1,091 (D) 1,119 (E) 1,116

18. The area of rectangle ABCD is 30. It is divided into 5 congruent rectangles, as seen in the adjoining diagram. Find the perimeter of ABCD.



(A) 22 (B) 32 (C) 34 (D) 38
(E) cannot be determined from information given

19. At what time after 5:30 will the hands of a clock be perpendicular?

(A) 5:40 (B) 5:42 (C) 5:43 (D) $5:43\frac{7}{11}$ (E) $5:43\frac{9}{11}$

20. In a factory, 80% of the unskilled workers are men, 90% of the skilled workers are men, and 82% of all workers combined are men. The ratio of the number of skilled workers to the number of unskilled workers is

(A) 1:1 (B) 1:2 (C) 1:3 (D) 1:4 (E) 1:5

MATHEMATICS TEST 11
Time Limit: 30 Minutes

1. Which of the following is equal to $\frac{(12)(-21)}{(-6)(-1)(-7)(-4)(-3)}$?

(A) 2 (B) -2 (C) $\frac{1}{2}$ (D) $\frac{-1}{2}$ (E) 1

2. Which of the following has the smallest value?

(A) .1 (B) $(.2)^3$ (C) $\sqrt{.25}$ (D) .05 (E) $\frac{1}{100}$

3. Which of the following fractions is closest in value to $\frac{4}{7}$?

(A) $\frac{5}{9}$ (B) $\frac{7}{12}$ (C) $\frac{6}{11}$ (D) $\frac{3}{5}$ (E) $\frac{1}{2}$

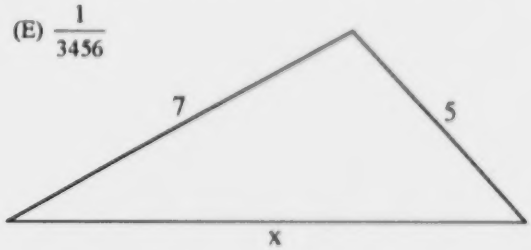
4. If Freddy walks 15 miles in 5 hours, by how many miles per hour must he increase his rate in order to walk the same distance in 3 hours?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

5. If 1 cubic inch of lead weighs approximately $\frac{1}{2}$ ounce, what part of a cubic foot is 1 ounce of lead?

(A) $\frac{1}{144}$ (B) $\frac{1}{288}$ (C) $\frac{1}{864}$ (D) $\frac{1}{1728}$ (E) $\frac{1}{3456}$

6. What values may x take on in the adjoining diagram?



(A) $x > 2$ (B) $x < 12$ (C) $x > 0$
(D) $2 \leq x \leq 12$ (E) $2 < x < 12$

7. The side of a square is s; each side of a triangle is t. If their perimeters are equal, express s in terms of t.

(A) $\frac{4t}{3}$ (B) $\frac{3s}{4}$ (C) $4t = 3s$ (D) $4s + 3t$ (E) $\frac{3t}{4}$

8. If $\frac{5}{8}$ of a gallon of soap costs \$2.75, how much will $\frac{3}{5}$ of a gallon of the same soap cost?

(A) \$2.60 (B) \$1.65 (C) \$1.72 (D) \$2.50 (E) \$2.64

9. In a certain state, a car is taxed according to its horsepower: 30¢ for each of the first 24 horsepower, and 40¢ for each succeeding horsepower. Find the horsepower of a car whose tax is \$10.00.

(A) 7 (B) $24\frac{1}{4}$ (C) 25 (D) 28 (E) 31

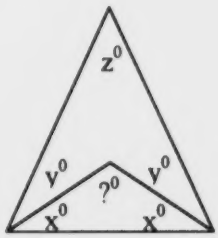
10. If $\frac{3}{4}$ of the voters in an election voted for Candidate A and $\frac{4}{5}$ of the remaining votes were for Candidate B, what fraction of the votes were for the other candidates?

(A) $\frac{3}{5}$ (B) $\frac{1}{5}$ (C) $\frac{9}{20}$ (D) $\frac{1}{9}$ (E) $\frac{1}{20}$

11. Find the next three terms in the following sequence: 0, 1, 2, 3, 6, 7, 14, 15, 30, ___, ___, ___, . . .

(A) 31, 32, 64 (B) 31, 62, 63 (C) 31, 32, 33 (D) 45, 46, 92
(E) 46, 47, 94

12. In the adjoining figure, express the measure of the "?" angle in terms of x, y, and z.



(A) 2x + 2y (B) 2x + 2y + z (C) 2y + z (D) x + y + z (E) 2z + x

13. If, for every number "a" and every number "b," a*b means a + 3b, select the false statement:

(A) The value of 3*4 is 15 (B) $2*(3*4) \neq (2*3)*4$ (C) $2*3 \neq 3*2$ (D) The operation is not associative
(E) The operation is commutative

14. A cow is tied by a 20-foot rope to a post placed at the intersection of two fences, each 300 feet long. If the two fences meet at their centers at an angle of 130^0, how many square feet are there in the greater area that the cow can roam?

(A) 400 (B) 620 (C) 400π (D) 620π (E) $\frac{1300\pi}{9}$

15. If two sides of a triangle are each 2x units long and the altitude drawn to the third side is x units long, which of the following represents the length of the third side of the triangle?

(A) 2x (B) 3x (C) $x\sqrt{3}$ (D) $2x\sqrt{3}$ (E) 5x

16. On a scale drawing of a form, an eighth of an inch corresponds to one rod. A field of 20 acres is represented on the drawing by an area of how many square inches? (1 acre = 160 square rods.)

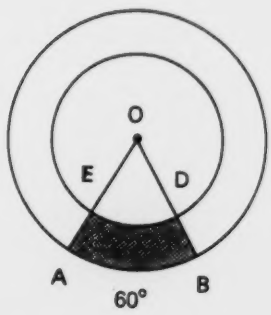
(A) 100 (B) 50 (C) 32 (D) 400 (E) 200

17. If 15 typists can type 240 pages in 8 hours, how long will it take 16 typists to type 384 pages at the same rate?

(A) 8 (B) 10 (C) 12 (D) 15 (E) none of these

18. In the adjoining diagram, the two circles are concentric.
Radius OA = 8 inches, DB = 2 inches, and the measure of arc AB = 60^0.
How many square inches are in the area of the shaded region?

(A) $4\frac{1}{3}\pi$ (B) $4\frac{2}{3}\pi$ (C) 6π (D) 64π (E) none of these



Use the following information for Nos. 19 and 20:

In Germany's monetary system: 100 Pfennigs = 1 D. Mark
In Austria's monetary system: 100 Groschen = 1 Schilling
Also: 4 D. Marks is equivalent to 1 U.S. dollar
25 Schillings is equivalent to 1 U.S. dollar

19. If an article costs 3 D. Marks and 60 Pfennigs, find the cost of this same article in Austrian currency.

(A) 22 Schillings and 50 Groschen (B) 360 Schillings (C) 90 Schillings
(D) 21 Schillings and 75 Groschen (E) none of these

20. If an article costs 192 Schillings and 50 Groschen, find the cost of this same article in German currency.

(A) 7 D. Marks and 70 Pfennigs (B) 28 D. Marks (C) 48 D. Marks and 12 Pfennigs
(D) 30 D. Marks and 80 Pfennigs (E) none of these

MATHEMATICS TEST 12
Time Limit: 30 Minutes

1. Reshelve these library books in increasing order of their Dewey Decimal numbers:

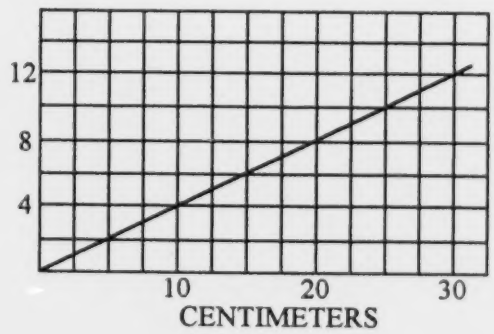
[1] 971.004956 [2] 971.00495 [3] 971.00495611 [4] 971.00495519
(A) 2, 3, 4, 1 (B) 2, 4, 3, 1 (C) 3, 2, 4, 1 (D) 2, 4, 1, 3 E) 2, 3, 4, 1

2. Suppose the sum of two two-digit numbers is a three-digit number. What is the first digit of the sum?

(A) 1 (B) 2 (C) 3 (D) 4 (E) cannot be determined with information given

3. This is a conversion graph for changing inches to centimeters. Find the number of centimeters equivalent to 5".

(A) 15 (B) $12\frac{1}{2}$ (C) 2 (D) 1 (E) none of these



4. If 1 bushel equals 4 pecks, and 1 peck equals 8 quarts, what percent of $\frac{1}{4}$ bushel are 10 quarts?

(A) 50% (B) 60% (C) 80% (D) 120% (E) 125%

5. A ramp 25 yards long rises 5 yards. Walter stops after pushing his wagon 20 yards up this ramp. How many yards has he risen?

(A) 1 (B) $2\frac{1}{5}$ (C) 3 (D) 4 (E) $4\frac{1}{5}$

6. If the measure of one angle of a triangle equals the sum of the measures of the other two angles, then the triangle must be

(A) right (B) acute (C) obtuse (D) isosceles (E) scalene

7. A car owner finds that she needs 1 quart of oil for each 120 miles she drives. If she has new rings put in, she will need only 80% as much oil. How many miles will a quart then last her?

(A) 150 (B) 96 (C) 144 (D) 90 (E) 160

8. A motor-powered model airplane is operated at the end of a cord. If it completes a circular circuit in $1\frac{1}{4}$ minutes, through how many degrees does it travel in a second?

(A) 2.88 (B) 5.9 (C) $4\frac{4}{5}$ (D) 4.5 (E) none of these

9. If it takes a plane 5 hours to travel 1000 miles, by how many miles per hour would it have to increase its rate to make the trip in $4\frac{1}{2}$ hours?

(A) $222\frac{2}{9}$ (B) $1\frac{1}{9}$ (C) $22\frac{2}{9}$ (D) 100 (E) none of these

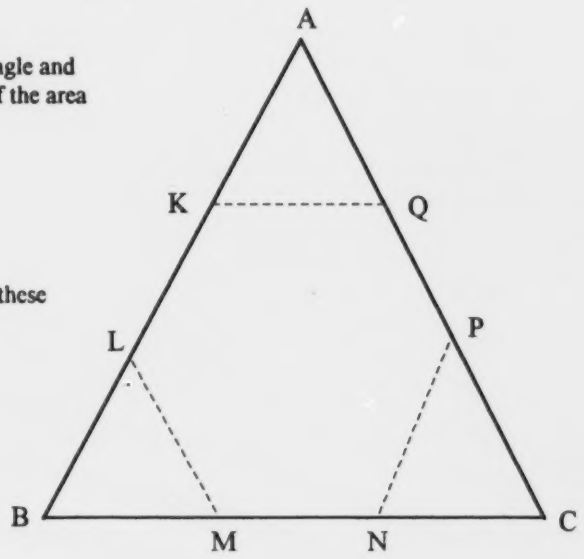
10. The cost of Puzimobiles is 18 dollars each. The selling price is S dollars each. What is the percent of profit on the selling price?

(A) $\frac{100S - 18}{S}$ (B) $\frac{100(S - 18)}{S}$ (C) $\frac{S - 18}{S}$ (D) $\frac{18 - S}{100S}$ (E) $\frac{18}{100S}$

11. In the adjoining figure, ABC is an equilateral triangle and KLMNPQ is a regular hexagon. What is the ratio of the area of ΔABC to the area of hexagon KLMNPQ?

(Note: $KL = \frac{1}{3} AB$.)

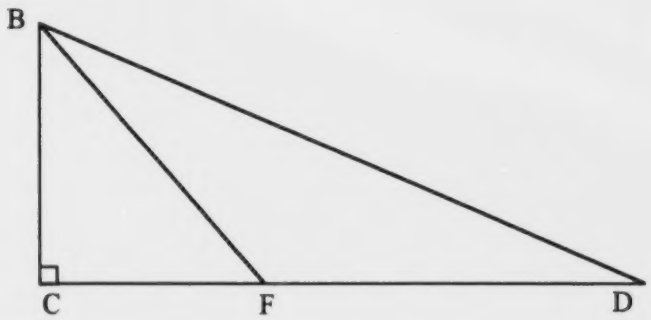
(A) $\frac{3}{1}$ (B) $\frac{9}{4}$ (C) $\frac{3}{2}$ (D) $\frac{5}{4}$ (E) none of these



12. In this diagram, $m\angle C$ is 90^0; CB = 12; DC = 27; DF = $1\frac{1}{4}$ CF.

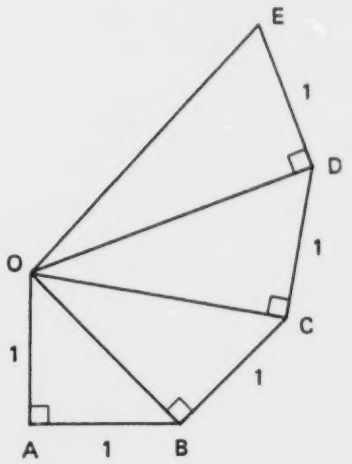
Find the measure of $\angle FBC$.

(A) 45^0 (B) 30^0 (C) 50^0 (D) 60^0
(E) impossible to find



13. In the adjoining diagram, with right angles marked at A, B, C, and D, OA = AB = BC = CD = DE = 1. Find the length of OE.

(A) $2\sqrt{2}$ (B) $3\sqrt{3}$ (C) $4\sqrt{3}$
(D) $\sqrt{5}$ (E) $\sqrt{6}$



14. To be on the honor roll in High School "X," Lynn must achieve an average of 90% in all her courses. So far this term Lynn has earned grades of 87% in history, 93% in English, 96% in chemistry, 85% in French, and 88% in music. What must her mark in her mathematics course be so that she may be placed on the honor roll?

(A) 89% (B) $89\frac{1}{2}\%$ (C) 90% (D) 91% (E) 92%

15. While shopping, David is faced with the following problem. Should he shop in store "A," which offers him successive discounts of 10% and 20%, or should he shop at store "B" which offers him successive discounts of 15% and 15%? Which of the following statements is correct?

(A) The discount at "A" exceeds the discount at "B" by $\frac{3}{4}\%$.

(B) The discount at "B" exceeds the discount at "A" by $\frac{3}{4}\%$.

(C) The discount at "A" exceeds the discount at "B" by $\frac{1}{4}\%$.

(D) The discount at "B" exceeds the discount at "A" by $\frac{1}{4}\%$.

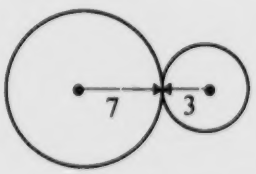
(E) The discount at "A" equals the discount at "B."

16. Edith sells 13 containers of milk daily; each container holds 22 quarts. A quart of milk weighs $1\frac{1}{2}$ pounds. If each of Edith's cows gives 25 pounds of milk each day, how many cows must she have at least?

(A) 12 (B) 17 (C) 18 (D) 21 (E) 23

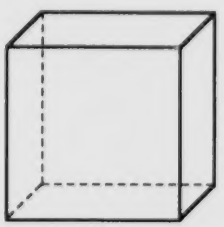
17. $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ is defined to be equal to ad - bc, in which a, b, c, and d are any numbers. Find x if $\begin{vmatrix} 3 & 7 \\ x & -5 \end{vmatrix} = 6$.

(A) 3 (B) -3 (C) $\frac{9}{7}$ (D) $\frac{-9}{7}$ (E) -2



18. Two meshed gears have radii of 7 inches and 3 inches as shown in the adjoining diagram. How many times must the smaller gear be turned in order for the arrows to be lined up again?

(A) 1 (B) 2 (C) 3 (D) 5 (E) 7



19. The adjoining diagram represents a hollow cube with 2-inch edges. A bug, inside the cube, begins to walk along the inside edges of the cube. What is the greatest number of inches that the bug can travel, never traveling along the same edge twice?

(A) 8" (B) 12" (C) 16" (D) 18" (E) 24"

20. A dinner check was to be divided equally among 8 people. When one refused to pay, each of the others had to pay 20¢ more. What was the total amount of the check?

(A) \$9.80 (B) \$ 11.20 (C) \$ 12.80 (D) \$ 14.40 (E) \$ 16.20

MATHEMATICS TEST 13
Time Limit: 30 Minutes

1. 5 is $\frac{1}{8}$% of which number?

(A) .4 (B) 40 (C) 50 (D) 250 (E) 4000

2. Evaluate: $\frac{1 \cdot 2 \cdot 3 \cdot 4 + 5 \cdot 6 \cdot 7 \cdot 8}{5 \cdot 6 \cdot 7 \cdot 8}$

(A) $\frac{71}{70}$ (B) $\frac{54}{5}$ (C) 24 (D) 12 (E) none of these

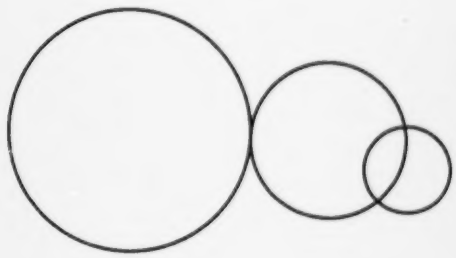
3. If $a > b$ and $b > 0$, which statement defines all possible values of c if $ac < 0$?

(A) $c < 0$ (B) $c > 0$ (C) $c \neq 0$ (D) $c > a$ (E) $c < b$

4. A diagram is copied using a 3:2 scale; that is, a 2" line becomes 3", and so on. By what percent is the area of the original increased in the copy?

(A) 50 (B) 250 (C) 150
(D) 225 (E) 125

5. In the adjoining diagram, the number of common tangents, taking two circles at a time, is



(A) 6 (B) 8 (C) 9 (D) 11 (E) 12

6. The pressure, P, of steam varies with the temperature, t, according to the formula $P = rt + k$, in which r and k are constants. If the pressure is 41 pounds at 270^0 and 49 pounds at 280^0, what is the value of r?

(A) $\frac{9}{55}$ (B) $\frac{11}{63}$ (C) $\frac{4}{5}$ (D) $\frac{6}{5}$ (E) 9

7. The greatest number of cubical blocks, two inches on each edge, which can be put into a cubical box 2 feet on each edge is

(A) 1600 (B) 1728 (C) 8 (D) 216 (E) 532

8. A shirt originally priced at \$2.50 was sold for \$1.98. By what percent was the price of the shirt reduced?

(A) 21% (B) 52% (C) 26% (D) 20.8% (E) 19.98%

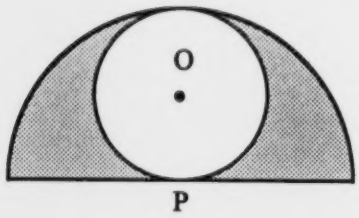
9. There were 396 people in a theater. If the ratio of women to men was 2:3, and the ratio of men to children was 1:2, how many men were there in the theater?

(A) 36 (B) 72 (C) 108 (D) 132 (E) 198

10. If x is a positive number, which of the following cannot be the average of 3, 6, 9, 12, and x?

(A) 6 (B) 9 (C) 11 (D) 12 (E) 15

11. In the adjoining diagram, circle O, of radius 2, is the largest circle which may be inscribed in the semicircle. Find the area of the shaded region.



(A) $\frac{\pi}{2}$ (B) π (C) 2π (D) 3π (E) 4π

12. To cover the six sides of a cube, $\frac{1}{2}$ quart of paint is needed. If two such cubes are placed next to each other and glued together to form one rectangular solid, how many quarts of paint will now be needed to cover this newly formed rectangular solid?

(A) $\frac{5}{3}$ (B) $\frac{6}{5}$ (C) $\frac{5}{6}$ (D) 1 (E) 2

13. Evaluate: $\sqrt{\frac{n^2}{36} + \frac{n^2}{64}}$

(A) $\frac{5n}{24}$ (B) $\frac{7n}{24}$ (C) $\frac{7n}{12}$ (D) $\frac{5n}{12}$ (E) none of these

14. What is the maximum number of points of intersection of 5 unequal circles?

(A) 16 (B) 17 (C) 18 (D) 20 (E) 25

15. A certain solution contains P% sugar. How much water must be evaporated from G gallons of it to produce a solution that is S% sugar?

(A) $\frac{PG}{100}$ (B) $\frac{S}{G(P - S)}$ (C) $\frac{G(P - S)}{S}$ (D) $\frac{G(S - P)}{S}$ (E) none of these

16. A f

ur coat was marked to sell at a profit of $66\frac{2}{3}\%$ on the cost. It was sold for \$540.00 at a profit of $33\frac{1}{3}\%$. The purchaser received a rate which represented a discount on the marked price of

(A) 10% (B) 20% (C) $33\frac{1}{3}\%$ (D) 50% (E) 75%

17. Al and Ernie together have 14 marbles. Ernie and Max together have 10 marbles. Al and Max together have 12 marbles. What is the maximum number of marbles that any one of these boys can have?

(A) 7 (B) 8 (C) 9 (D) 10 (E) 12

18. While shopping for a computer, Lynn is faced with a decision. Should she ask for a $12\frac{1}{2}\%$ discount on the store's 20% discount sale, or should she merely ask the store clerk for a 32% discount on the regular price? What is the difference between the two discounts?

(A) the first exceeds the second by $\frac{1}{2}\%$

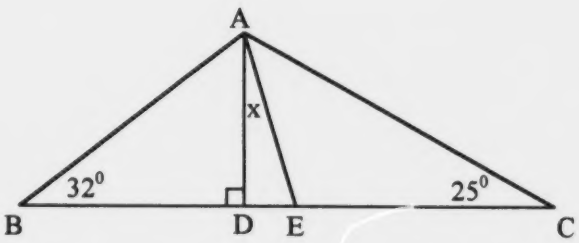
(B) the second exceeds the first by 2%

(C) the first exceeds the second by $2\frac{1}{2}\%$

(D) both discounts are the same

(E) cannot be determined with the information given

19. In the adjoining ΔABC, $m\angle B = 32^0$, and $m\angle C = 25^0$. Find $m\angle x$ (between the altitude $\overline{AD}$ and the bisector $\overline{AE}$ of $\angle BAC$).



(A) $3\frac{1^0}{2}$ (B) $4\frac{1^0}{2}$ (C) $7\frac{1^0}{2}$ (D) 41^0 (E) $28\frac{1^0}{2}$

20. How many minutes after 2 o'clock will the minute hand of a clock overtake the hour hand?

(A) 10 (B) $10\frac{10}{11}$ (C) 11 (D) $11\frac{1}{11}$ (E) $11\frac{5}{11}$

MATHEMATICS TEST 14
Time Limit: 30 Minutes

1. Where does $\frac{3}{8}$ the distance from $-2\frac{1}{2}$ to $1\frac{1}{3}$ lie on the number line?

(A) $-2\frac{1}{8}$ (B) $-1\frac{5}{16}$ (C) $-1\frac{1}{16}$ (D) $\frac{1}{16}$ (E) $-\frac{23}{24}$

2. If $\frac{1}{4}$% of a solution is acid, what fractional part of the solution is not acid?

(A) $\frac{3}{4}$ (B) $\frac{1}{4}$ (C) $\frac{1}{400}$ (D) $\frac{399}{400}$ (E) $\frac{4}{100}$

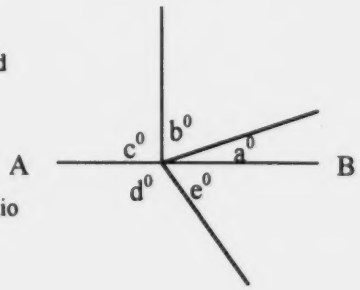
3. 60% of a foot is how many twelfths of a yard?

(A) 2 (B) $2\frac{1}{5}$ (C) $2\frac{2}{5}$ (D) $2\frac{3}{5}$ (E) 3

4. A truck of type A can carry a load 40% heavier than one of type B. If 20 type B trucks are needed to carry a certain load, how many type A trucks can do the same job?

(A) 14 (B) 28 (C) 24 (D) 12 (E) 15

5. In the adjoining diagram, AB is a straight line and a:b:c:d = 1:2:3:4. Find the measure of $\angle e$.



(A) 15^0 (B) 30^0 (C) 45^0 (D) 60^0 (E) 120^0

6. If two circular cans have the same volume and their heights are in the ratio 5:4, then their radii are in the ratio:

(A) $\sqrt{5}:3$ (B) $\sqrt{5}:4$ (C) 4:5 (D) $\sqrt{2}:\sqrt{5}$ (E) $2:\sqrt{5}$

7. Consider the following ordered set: (6, 7, 8, x). If the average of the first 3 elements is equal to x, then the average of all four elements is

(A) 7 (B) $7\frac{1}{2}$ (C) $7\frac{3}{4}$ (D) $7\frac{7}{8}$ (E) none of these

8. If $x = 2 + 2k$, and $x \neq 0$, then $\frac{1}{x} + \frac{1}{x} + \frac{1}{x} + \frac{1}{x} = ?$

(A) $\frac{1}{k}$ (B) $\frac{1}{1+2k}$ (C) $\frac{2}{1+k}$ (D) $\frac{1}{1+k}$ (E) $\frac{2}{2+k}$

9. If (2, 4) are the coordinates of one vertex of a square whose area is 25, which of the following could be the vertex diagonally opposite the given vertex?

(A) (-3, 0) (B) (5, 0) (C) (5, -1) (D) (7, -1) (E) none of these

10. In Anyville there are 48,000 TVs, a number that represents 12.8 TVs per 100 persons. What is the population of Anyville?

(A) 614,400 (B) 500,000 (C) 375,000 (D) 128,000 (E) none of these

11. If $a = \frac{np}{n+1}$, then $n = ?$

(A) $\frac{a}{a-p}$ (B) $\frac{a}{p-a}$ (C) $\frac{p-a}{a}$ (D) $\frac{a-p}{a}$ (E) $\frac{a}{p+a}$

12. A cube with 3-inch edges is painted blue. It is then cut into 1-inch cubes with 6 cuts. How many of these 1-inch cubes have only 2 blue faces?

(A) 6 (B) 8 (C) 10 (D) 12 (E) 24

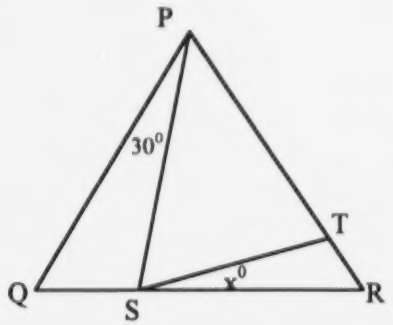
13. A 12-quart solution of acid and water is 10% acid. If 4 quarts of water are added, the solution becomes _____ acid.

(A) 50% (B) 40% (C) $33\frac{1}{3}\%$ (D) 75% (E) 7.5%

14. If a pipe with diameter a carries only $\frac{3}{4}$ of the necessary amount of water, then a pipe that will carry just the necessary amount of water must have a diameter of

(A) $\frac{4}{3}a$ (B) $\frac{8}{3}a$ (C) $\frac{2a\sqrt{3}}{3}$ (D) $\frac{a}{\sqrt{3}}$ (E) $\frac{a\sqrt{3}}{4}$

15. In the adjoining diagram, PQ = PR and PS = PT. If $m\angle QPS = 30^0$, find $m\angle x$.

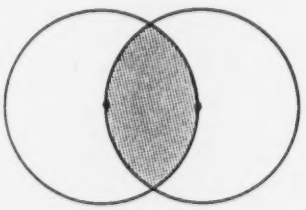


(A) 10^0 (B) 15^0 (C) 20^0 (D) $7\frac{1^0}{2}$ (E) $22\frac{1^0}{2}$

16. An efficiency expert, paid \$24 per day, is employed by a company that pays 450 employees \$1.25 an hour each. If she is able to save 3 man-hours per employee per day, how much does she save the company each day?

(A) \$537.50 (B) \$351 (C) \$187.50 (D) \$163.50 (E) \$155

17. In the adjoining diagram, the two equal circles of radius 6 pass through each other's center. Find the area of the shaded region.



(A) 12π (B) $6\pi - 9\sqrt{3}$ (C) $12\pi - 18\sqrt{3}$ (D) $24\pi - 18\sqrt{3}$
(E) $36\pi - 18\sqrt{3}$

18. A 25-foot ladder is placed against a vertical wall of a building. The foot of the ladder is 7 feet from the base of the building. If the top of the ladder slips 4 feet, then the foot of the ladder will slide

(A) 9 feet (B) 15 feet (C) 5 feet (D) 8 feet (E) 4 feet

19. In a triangle with sides of length 13, 14, and 15, find the length of the altitude to the side of length 14.

(A) 8 (B) 10 (C) 12 (D) 14 (E) cannot be determined with the information given

20. A fish has a head 7 inches long. Its tail is as long as its head and one half as long as its body. The length of its body equals the length of its head and tail together. Find the length of the entire fish.

(A) 21 (B) 28 (C) 42 (D) 56 (E) 63

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MATHEMATICS TEST 15
Time Limit: 30 Minutes

1. $\frac{5}{8} + \frac{1}{x} = \frac{34}{48}$. What is the value of x?

(A) 4 (B) 8 (C) 12 (D) 24 (E) 48

2. Which of the following statements is true about the set of numbers $\left\{.75, -\frac{1}{2}, \frac{1}{2}\right\}$?

(A) $.75 < -\frac{1}{2} < \frac{1}{2}$ (B) $-\frac{1}{2} < .75 < \frac{1}{2}$ (C) $\frac{1}{2} < -\frac{1}{2} < .75$ (D) $\frac{1}{2} < .75 < -\frac{1}{2}$ (E) none of these

3. If a brick weighs 5 pounds and a half of a brick, then the weight of the brick expressed in pounds is

(A) 6 (B) 7 (C) $7\frac{1}{2}$ (D) 9 (E) 10

4. If Mac makes a profit of 20% on the selling price of a hamburger, what per cent profit does he make on the cost?

(A) 20% (B) 25% (C) 40% (D) 80% (E) none of these

5. Ernie buys 100 4¢ stamps and 100 5¢ stamps. At the same time, Alice buys 500 4¢ stamps and 500 5¢ stamps. Which of the following represents the difference between the average spent per stamp by Ernie and Alice?

(A) 0¢ (B) 4¢ (C) $12.00 (D) $ 13.50 (E) $ 15.00

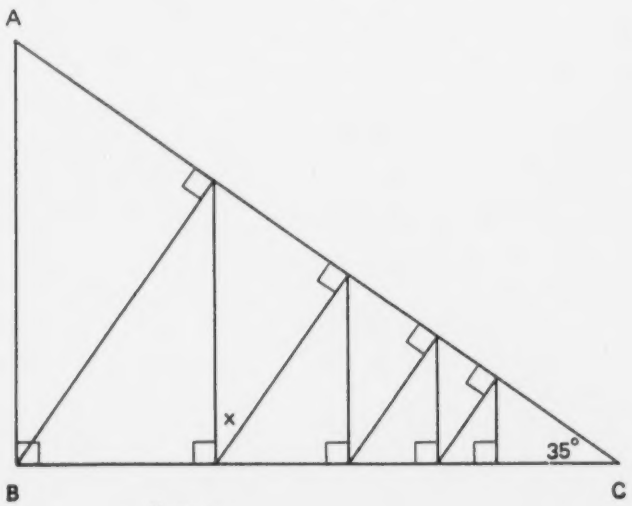
6. If $\frac{1}{x} = \frac{1}{a} + \frac{1}{b}$,then the value of x is

(A) $a + b$ (B) $\frac{2ab}{a + b}$ (C) $\frac{a + b}{ab}$

(D) $\frac{ab}{a + b}$ (E) ab

7. In the adjoining diagram, all the perpendicular lines are marked as in right $\angle ABC$. $m\angle ACB = 35^0$. Find the measure of $\angle x$.

(A) 30^0 (B) 35^0 (C) 45^0 (D) 55^0
(E) 60^0



8. Find x in the following sequence: 1, 1, 2, 3, 5, 8, x, 21, 34, 55, . . .

(A) 11 (B) 12 (C) 13 (D) 14 (E) 15

9. The size of a new camera is 25% smaller than the size of the old one, and it weighs $\frac{1}{2}$ as much. The weight per cubic unit of the new camera is how many times that of the old camera?

(A) 2 (B) $\frac{2}{3}$ (C) $\frac{3}{2}$ (D) $\frac{1}{8}$ (E) $\frac{3}{8}$

10. $62\frac{1}{2}\%$ of the selling price of a suit covers the cost of manufacturing it; $33\frac{1}{3}\%$ pays for sales expenses; the remaining \$1.50 is profit. Find the cost of manufacturing the suit.

(A) \$36 (B) \$22.50 (C) \$45 (D) \$12 (E) none of these

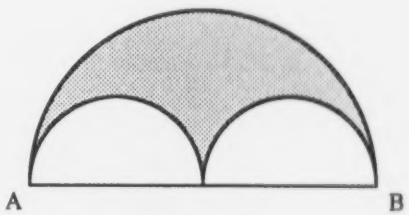
11. In selling a boat, Bert expected to gain 10% on the cost price. However, he received \$50 less than he expected, and thus lost \$15 on the cost price. The sale price, in dollars, was

(A) 220 (B) 235 (C) 250 (D) 335 (E) 350

12. A racing auto goes 4 laps at the rate of 50 seconds per lap, and then it averages a lap per minute for the last 6 laps. What is its average time per lap?

(A) 54 (B) 55 (C) $55\frac{1}{2}$ (D) 56 (E) $56\frac{1}{2}$

13. The shaded region in the adjoining figure is formed by three tangent semicircles, two of which are congruent. If the shortest distance from point A to point B is 12, find the area of the shaded region.



(A) 9π (B) $13\frac{1}{2}\pi$ (C) 18π (D) 27π (E) 72π

14. If the area of a square is 72 square inches, then the area of the circumscribed circle in square inches is

(A) 24π (B) 32π (C) 36π (D) 48π (E) 72π

15. Evelyn is 3 years younger than Henry. The sum of their ages is greater than 30 and less than 35. Henry's age is

(A) 19 (B) either 18 or 17 (C) 16 (D) either 15 or 14 (E) 13

16. Each wheel of a wagon has a diameter of 3 feet. How many feet will the wagon move forward when each wheel turns through 50^0 without slipping? (assume $\pi = \frac{22}{7}$)

(A) $\frac{11}{84}$ (B) $\frac{55}{84}$ (C) $\frac{55}{42}$ (D) $\frac{55}{21}$ (E) $\frac{33}{7}$

17. Two boys match pennies. If one boy loses, he gives the other boy one penny. When the boys finish, one of the boys has won three times, and the other has seven more pennies than when he started. The number of times the boys matched pennies was

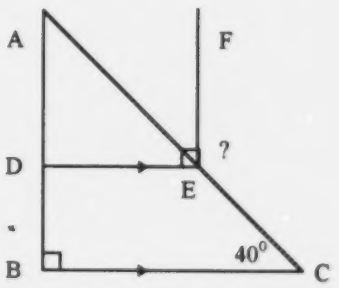
(A) 13 (B) 10 (C) 17 (D) 7 (E) 11

18. There were 5 candidates, A, B, C, D, and E, in a class election. A was elected with 26 votes. B placed second. E came in last with 2 votes. If no two candidates had the same number of votes, and there were 45 votes cast, what is the smallest number that B could have received?

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

19. In the adjoining diagram, $\overline{DE}$ is parallel to $\overline{BC}$, and $\overline{DE}$ is perpendicular to $\overline{FE}$. If $m\angle ACB = 40^0$, find the degree measure of $\angle FEC$.

(A) 50^0 (B) 125^0 (C) 130^0 (D) 140^0
(E) cannot be determined with information given



20. If, on the average, a hen and a half can lay an egg and a half in a day and a half, how many eggs should six hens lay in 8 days?

(A) 8 (B) $8\frac{1}{2}$ (C) 24 (D) 32 (E) none of these

MATHEMATICS TEST 16
Time Limit: 30 Minutes

1. The cost of 100 pencils is C cents. At this rate, the number of pencils that can be bought for S cents is:

(A) $\frac{C}{100S}$ (B) $\frac{CS}{100}$ (C) $\frac{100S}{C}$ (D) $\frac{S}{100C}$ (E) $\frac{100C}{S}$

2. If a 5-foot bar of steel weighs 9.0 pounds, how many pounds does a 10-inch bar of the same kind weigh?

(A) 1.5 (B) 1.8 (C) 18 (D) 15 (E) 14

3. If an airplane flies 550 yards in 3 seconds, the airplane's speed in miles per hour is

(A) 300 mph (B) 375 mph (C) 125 mph (D) 275 mph (E) none of these

4. Given the set A = {1, 2, 3, 4, 5, 8, 9, 10, 12, 16} and the set B = {divisors of 36}, which of the following sets contains all the numbers belonging to both A and B?

(A) {2, 3, 4, 9} (B) {2, 3, 9, 12} (C) set A - {5, 8, 10, 16} (D) {2, 4, 6, 12} (E) none of these

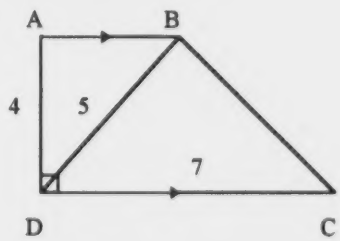
5. Solve the following equation for x: $2 = \frac{1 - \frac{x}{y}}{3}$

(A) 6y - 1 (B) 1 - 6y (C) $\frac{y - x}{3}$ (D) 5y (E) -5y

6. One end of a ladder 32 feet long is placed 10 feet from the outer wall of a building that stands vertically on level ground. How far up the building, TO THE NEAREST FOOT, will the ladder reach?

(A) 28 feet (B) 29 feet (C) 30 feet (D) 31 feet (E) none of these

7. In the adjoining diagram, quadrilateral ABCD has $\overline{AB}$ parallel to $\overline{DC}$, and $\overline{AD}$ perpendicular to $\overline{DC}$. If AD = 4, BD = 5, and DC = 7, find the area of ΔBDC.



(A) 6 (B) 8 (C) 14 (D) $17\frac{1}{2}$
(E) cannot be determined with information given

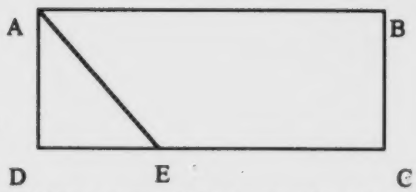
8. Judy drove 175 miles at 25 miles per hour. If she had driven 10 miles per hour faster, how many hours would she have saved?

(A) $1\frac{1}{2}$ (B) $17\frac{1}{2}$ (C) $10\frac{1}{2}$ (D) 2 (E) none of these

9. If we define the operation * for positive numbers to be $a * b = \frac{ab}{a + b}$, then 8 * (8 * 8) equals

(A) $\frac{3}{8}$ (B) $\frac{8}{9}$ (C) $2\frac{2}{3}$ (D) 4 (E) $7\frac{1}{9}$

10. In the adjoining diagram, CE is $\frac{2}{3}$ of CD. The area of ΔADE is 21. Find the area of rectangle ABCD.



(A) 126 (B) 64 (C) 96 (D) 108 (E) cannot be determined with information given

11. At a "penny sale," you can buy three items at retail price and get a fourth for a penny. If Al buys twelve 10¢ cans of mixed vegetables, how much does he save during the sale?

(A) 3¢ (B) 27¢ (C) 30¢ (D) 93¢ (E) $1.20

12. In an election, 5219 votes were cast for 4 candidates. The winner exceeded his three opponents by 22, by 30, and by 73 votes, respectively. How many votes did the winner receive?

(A) 1336 (B) 1358 (C) 1366 (D) 1409 (E) none of these

13. If a bottle and a cork cost one dollar and five cents, and the bottle costs one dollar more than the cork, then the cost of the cork in cents is

(A) 2 (B) 3 (C) 4 (D) 5 (E) none of these

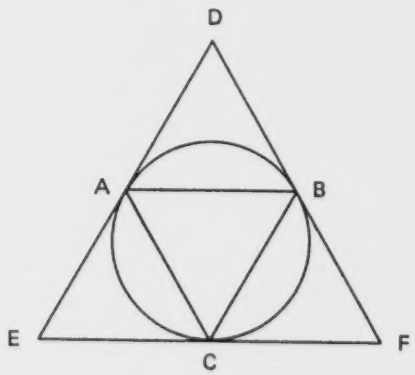
14. The formula $C = \frac{5}{9}(F - 32)$ shows the relationship between the centigrade and Fahrenheit temperature scales. When the centigrade (C) scale reads 20^0, what should the Fahrenheit (F) scale read?

(A) 7.54^0 (B) 4^0 (C) 68^0 (D) 80^0 (E) none of these

15. A rectangular box is such that the top has an area of 108 square inches; the front, an area of 84 square inches; and the side, an area of 63 square inches. Find the dimensions of the box.

(A) 6", 8", 9" (B) 7", 8", 9" (C) 12", 16", 9" (D) 7", 18", 9" (E) 12", 9", 7"

16. In the adjoining diagram, equilateral triangle ABC is inscribed in the circle and equilateral triangle DEF is circumscribed about the circle. If the difference of the areas of these two equilateral triangles is 24, which of the following represents the area of ΔABC?



(A) 6 (B) 8 (C) 12 (D) 16 (E) 24

17. There are enough peanuts in a bag to give 12 nuts to each of 20 children. However, 5 children do not want any. How many nuts can each of the others have?

(A) 3 (B) 9 (C) 16 (D) 25 (E) 48

18. If 20% of a 40-gallon solution is acid, then how many gallons of acid must be added to the solution to raise the acid content to 60%?

(A) 20 (B) 40 (C) 16 (D) 24 (E) 48

19. The average of the marks on a test, in which 75% of the class passed, was 70. If the average of the passing marks was 80, then the average of the failing marks was

(A) 40 (B) 45 (C) 50 (D) 60 (E) none of these

20. In a battle, 5% of an army were killed, $12\frac{1}{2}\%$ of the remainder were wounded, and $16\frac{2}{3}\%$ of the wounded died (i.e., were mortally wounded). If there were 290 more soldiers killed than mortally wounded, how many soldiers were in the army?

(A) 1,140 (B) 2,900 (C) 5,800 (D) 9,120 (E) 9,600

Quantitative Comparison Questions

Quantitative comparison questions emphasize the concepts of equalities, inequalities, and estimation. They generally involve less reading, take less time to answer, and require less computation than regular multiple-choice questions. Be careful not to mark answer option E when responding to the four-choice quantitative comparison questions.

Directions: Each of the following questions consists of two quantities in boxes, one in Column A and one in Column B. You are to compare the two quantities and on the answer sheet fill in oval

A if the quantity in Column A is greater
B if the quantity in Column B is greater
C if the two quantities are equal
D if the relationship cannot be determined from the information given

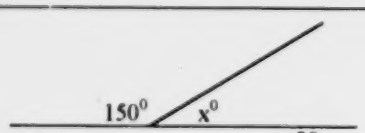
AN E RESPONSE WILL NOT BE SCORED

Notes:

1. In some questions, information is given about one or both of the quantities to be compared. In such cases, the given information is centered above the two columns and is not boxed.

2. In a given question, a symbol that appears in both columns represents the same thing in Column A as it does in Column B.

3. Letters such as x, n, and k stand for real numbers.

EXAMPLES	Column A	Column B	Answers
E1	5^2	20	■ B C D E
E2			A B ■ D E
	r and s are integers.		
E3	r + 1	s - 1	A B C ■ E

Explanations:

(The answer is A because 25 is greater than 20.)

(The answer is C because 150 + x = 180, thereby making x = 30.)

(The answer is D because not enough information is known about r and s.)

MATHEMATICS TEST 17

Time Limit: 30 Minutes

Quantitative Comparisons

***Directions*: Each question in this section consists of two quantities, one in Column A and one in Column B. You are to compare the two quantities and on the answer sheet blacken space**

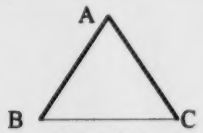
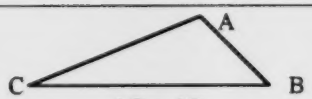
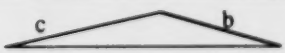
A if the quantity in Column A is the greater
B if the quantity in Column B is the greater
C if the two quantities are equal
D if the relationship cannot be determined from the information given

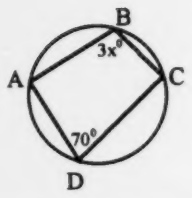
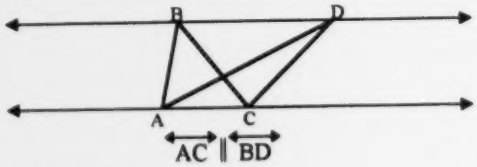
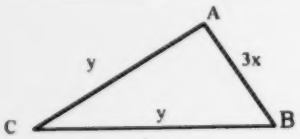
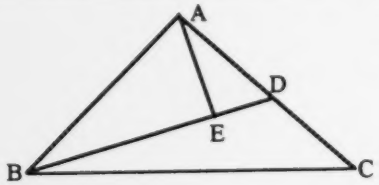
Notes: (1) In certain questions, information concerning one or both of the quantities to be compared is centered above the two columns. (2) A symbol that appears in both columns represents the same thing in Column A as it does in Column B. (3) All numbers in this test are real numbers. Letters such as x, n, and k stand for real numbers.

	Column A		Column B
1.	$8^{\frac{2}{3}}$		$32^{\frac{2}{5}}$
2.	$\frac{1}{x^3}$	$x < 0$	$\frac{1}{x^2}$
3.	-3	$-5x > 20$	x
4.	[-1.5]	[x] is the greatest integer less than or equal to 4	[-1]
5.	a	$\frac{a}{b} > 0$	b
6.	a + b		c
7.	x - s	$x = y$ and $s > t$	y - t
8.	$\frac{n}{x}$	$0 < y < x$ and $n < 0$	$\frac{n}{y}$
9.	2.2 meters	1 meter = 1.1 yards	2.2 yards

ANSWER CHOICES

A if the quantity in Column A is the greater
B if the quantity in Column B is the greater
C if the two quantities are equal
D if the relationship cannot be determined from the information given

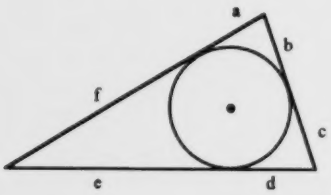
	Column A	Column B
10.	 AB = AC, $m\angle B > 45^0$	90^0
11.	 AC > AB	$m\angle C$
12.	$6(x - 10)^2 = 150$ x	16
13.	m = n + 1 $(m - n)^5$	$(n - m)^5$
14.	$y = \sqrt{x^2}$ y	-1
15.	Numbers a, b, c, and d are $\frac{a}{b}$ positive and ab > bc.	$\frac{c}{d}$
16.	x = -1 $x^5 + x^7$	$x^3 + x^4$
17.	The average of x, x + 1, x + 2	The average of x - 1, x, x + 1, x + 2, x + 3
18.	 a - b	c
19.	(10d + 1) dimes	(d + 10) dimes
20.	$\frac{-5}{\sqrt{5}}$	$-\sqrt{5}$
21.	The measure of the acute angle formed by the hands of a clock at 11:45	The measure of the acute angle formed by the hands of a clock at 3:25

	Column A	Column B
22.	 36	x
23.	 area ΔABC	area ΔADC
24.	 x	y
25.	 $m\angle BAE$	$m\angle BDC$
26.	$\lvert x \rvert = \lvert y \rvert$ x	y
27.	The average speed of 40 mph for 30 miles and 60 mph for 30 miles	50 mph
28.	$m\angle ACB = 90^0$ $\overline{AC}$ and $\overline{BC}$ are tangent to circle O at A and B, respectively 90^0	Measure of minor arc AB
29.	$x > y > 0$ $k > 0$ $\frac{x}{y}$	$\frac{x + k}{y + k}$

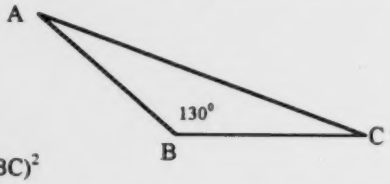
Column A **Column B**

PQ = 5 and PR = 6
O is the center of the circle
$\overline{OB} \perp \overline{PQ}$ and $\overline{OA} \perp \overline{PR}$

30. OB OA



31. $a + c + e$ $b + d + f$



32. $(AB)^2 + (BC)^2$ $(AC)^2$

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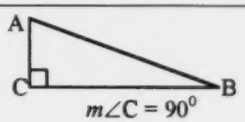
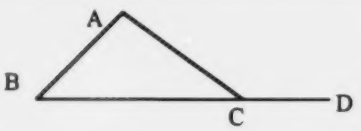
33. $|x + y|$ $|x| + |y|$

MATHEMATICS TEST 18
Time Limit: 30 Minutes
Quantitative Comparisons

***Directions*: Each question in this section consists of two quantities, one in Column A and one in Column B. You are to compare the two quantities and on the answer sheet blacken space**

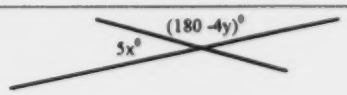
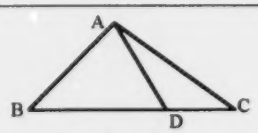
A if the quantity in Column A is the greater
B if the quantity in Column B is the greater
C if the two quantities are equal
D if the relationship cannot be determined from the information given

Notes: (1) In certain questions, information concerning one or both of the quantities to be compared is centered above the two columns. (2) A symbol that appears in both columns represents the same thing in Column A as it does in Column B. (3) All numbers in this test are real numbers. Letters such as x, n, and k stand for real numbers.

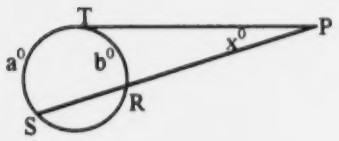
	Column A	Column B
	$7x = 0$	
1.	1	x
2.	(17)(18)(-5)(-3)(2)(-8)	(-12)(-1)(-2)(3)(-5)
	x and y are positive numbers, and $\frac{x}{y} > 1$.	
3.	x	y
	 $m\angle C = 90^0$	
4.	AB	AC
	1 meter = 3.3 feet	
5.	1.1 meters	4.4 feet
6.	$\frac{m}{17}$	$\frac{m}{19}$
		
7.	$m\angle A$	$m\angle ACD$

ANSWER CHOICES

A if the quantity in Column A is the greater
B if the quantity in Column B is the greater
C if the two quantities are equal
D if the relationship cannot be determined from the information given

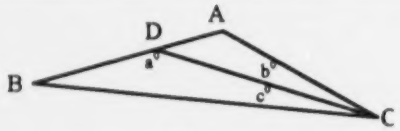
	Column A	Column B
	$n > 0$	
8.	The number of divisors of n	The number of divisors of n + 1
9.	The area of a nonrectangular parallelogram with sides 7 inches and 9 inches.	The area of a rectangle with sides 7 inches and 9 inches
	$7 - 12x > 5$	
10.	x	2
11.	$(x - y)^3$	$(y - x)^3$
	$x > a$ and $y > a$	
12.	x	y
13.	 x	y
14.	$(x + 8)(x - 3)$	$(x + 7)(x - 4)$
	$0 > a > b > c$	
15.	$\frac{c}{a}$	$\frac{c}{b}$
16.	$x (y - z)$	$-x (z - y)$
	$x = -2$	
17.	$\frac{x^5}{x^3}$	$\frac{x^7}{x^4}$
	 $BD > DC$	
18.	area ΔABD	area ΔADC
	$x^2 + y^2 = 8$ $xy = 3$	
19.	$(x + y)^2$	8

	Column A	Column B



$\overline{PT}$ is tangent to the circle at T.

20.	a - b	2x
21.	$\frac{1}{\sqrt{2} - 1}$	$1 + \sqrt{2}$

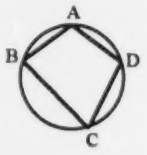


$\overline{DC}$ bisects $\angle ACB$

22.	a	c

$0 < a < b$ and $m < 0$

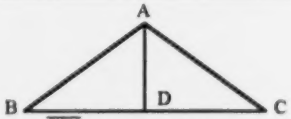
23.	$\frac{a}{b}$	$\frac{a - m}{b - m}$



24.	$m\angle A + m\angle C$	$m\angle B + m\angle D$

The surface area of a certain cube of edge length e is numerically greater than the volume.

25.	e	6

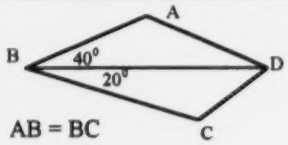


$\overline{AD}$ bisects $\angle BAC$

26.	area ΔABD	area ΔACD

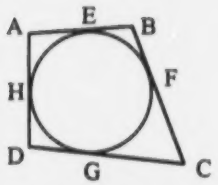
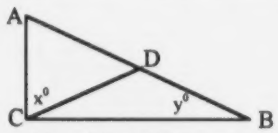
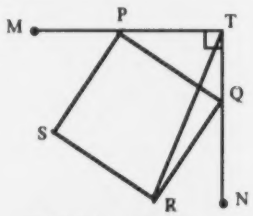
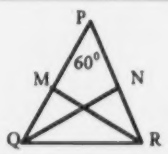
$y = 3m$ and m is a positive integer

27.	remainder of $\frac{56}{y}$	remainder of $\frac{58}{y}$



AB = BC

28.	AD	DC

	Column A	Column B
29.	 E, F, G, and H are points of tangency of the quadrilateral and the circle AB + DC	AD + BC
30.	The measure of the acute angle formed by the hand of a clock at 2:00.	The measure of the acute angle formed by the hand of a clock at 2:20.
31.	 AD = DB = CD 90^0	m∠ACB
32.	 PQRS is a square $\overline{MPT} \perp \overline{NQT}$ PT = 4, and QT = 3 TR	7
33.	 $\overline{RM} \perp \overline{PQ}$, $\overline{QN} \perp \overline{RP}$ $m\angle P = 60^0$, $m\angle PQR = 61^0$ QN	RM

MATHEMATICS TEST 19

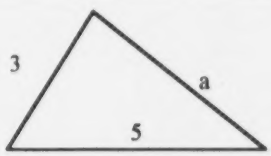
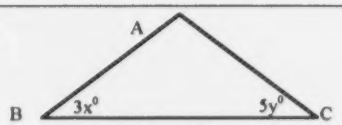
Time Limit: 30 Minutes

Quantitative Comparisons

***Directions*: Each question in this section consists of two quantities, one in Column A and one in Column B. You are to compare the two quantities and on the answer sheet blacken space**

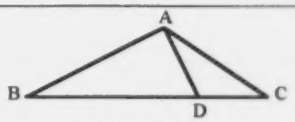
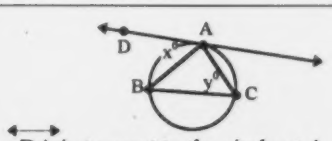
A if the quantity in Column A is the greater
B if the quantity in Column B is the greater
C if the two quantities are equal
D if the relationship cannot be determined from the information given

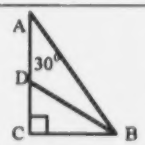
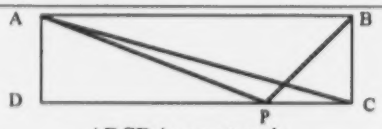
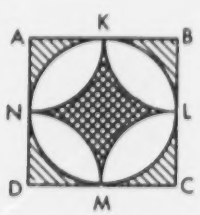
Notes: (1) In certain questions, information concerning one or both of the quantities to be compared is centered above the two columns. (2) A symbol that appears in both columns represents the same thing in Column A as it does in Column B. (3) All numbers in this test are real numbers. Letters such as x, n, and k stand for real numbers.

	Column A	Column B
	x and y are the next numbers in the sequences. 2, 3, 5, 8, 13, x 1, 2, 2, 4, 8, y	
1.	x	y
		
2.	a	2
	$p < 0$	
3.	$p^3 - 1$	-1
4.	$\frac{b - a}{a - b}$	$\frac{a - b}{b}$
	 AB = AC	
5.	x	y
	1 kilogram = 2.2 pounds	
6.	15 kilograms	33 pounds
	$x \neq 0$ and $y \neq 0$	
7.	$\frac{1}{x} + \frac{1}{y}$	$\frac{y + x}{xy}$
	$xm > ym$ and $m < 0$	
8.	x	y

ANSWER CHOICES

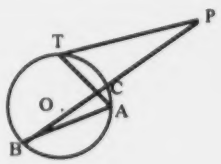
A if the quantity in Column A is the greater
B if the quantity in Column B is the greater
C if the two quantities are equal
D if the relationship cannot be determined from the information given

	Column A	Column B
9.	$2x + 5 < 5x - 1$ x	2
10.	a and b are positive numbers, and $\frac{a}{b} < 1$ a	b
11.	$x > 0$, $y \neq 0$, and $\frac{2}{x} = \frac{3}{y}$ x	y
12.	$x = -1$ x^{-5}	x^{-8}
13.	$a > b > 0$, and $c > 0$ $\frac{c}{a}$	$\frac{c}{b}$
14.	$a = b + 3$ The average of a - 2, a, a + 2	The average of b - 2, b + 5, b + 6
15.	$5(x - 7)^2 = 45$ x	5
16.	 AB = AC $m\angle ABD$	$m\angle ADB$
17.	 $\overleftrightarrow{DA}$ is tangent to the circle at A. x	y
18.	The positive integer n if $\sqrt{11} < n < \sqrt{20}$	The positive integer m if $\sqrt{3} < m < \sqrt{27}$

	Column A	Column B
	h is the length of the altitude of ΔABC from vertex A. m is the length of the median of ΔABC from vertex A.	
19.	h	m
	$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$	
20.	$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$	$\begin{vmatrix} 4 & 3 \\ 2 & 1 \end{vmatrix}$
21.	Area of a square with perimeter 36	Area of an equilateral triangle with perimeter 36
22.	The measure of the acute angle formed by the hands of a clock at 11:45	The measure of the acute angle formed by the hands of a clock at 8:30
23.	 $m\angle C = 90^0$ $m\angle A = 30^0$ $\overline{BD}$ bisects $\angle ABC$ DC	AD
24.	 ABCD is a rectangle Area ΔAPC	Area BPC
25.	 K, L, M and N are midpoints of the sides of square ABCD (respectively). NK, KL, ML and MN are each congruent 90^0 arcs of a circle. Area of region shaded	Area of region shaded

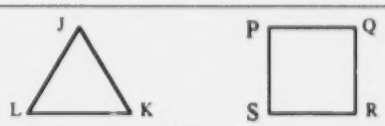
130

	Column A	Column B
	$y \neq 0, x > y, y \neq -k$	
26.	$\frac{x}{y}$	$\frac{x + k}{y + k}$
	$\lvert x - 8 \rvert < 6$	
27.	x	8
28.	The list price of an item	The amount paid for an item after an 8% discount on the list price and an 8% sales tax



$\overline{PT}$ is tangent to circle O

29.	$m\angle P$	$m\angle A$
30.	The average cost per gallon of gasoline for \$7.00 of 55¢ per gallon and \$7.00 of 65¢ per gallon	60¢ per gallon
	x and y are distinct, positive numbers	
31.	$\sqrt{xy}$	$\frac{x + y}{2}$



LK = SR

32.	Altitude of equilateral ΔJKL	Altitude of square PQRS
33.	The area of a square of perimeter p	The area of a nonsquare rectangle of perimeter p

MATHEMATICS TEST 20

Time Limit: 30 Minutes

Quantitative Comparisons

***Directions*: Each question in this section consists of two quantities, one in Column A and one in Column B. You are to compare the two quantities and on the answer sheet blacken space**

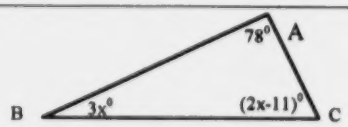
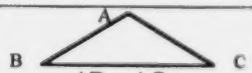
A if the quantity in Column A is the greater
B if the quantity in Column B is the greater
C if the two quantities are equal
D if the relationship cannot be determined from the information given

Notes: (1) In certain questions, information concerning one or both of the quantities to be compared is centered above the two columns. (2) A symbol that appears in both columns represents the same thing in Column A as it does in Column B. (3) All numbers in this test are real numbers. Letters such as x, n, and k stand for real numbers.

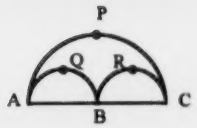
	Column A		Column B
1.	$a + c$	$a > b$ and $b > c$	$2b$
2.	0	$x = (-5)(-6)(-7)(-8)(-9)$	x
3.	BD	ABCD is a rectangle	DC
4.	x	$x^2 + 5x + 6 = 0$	0
5.	$\frac{\lvert n \rvert}{n}$	$n \neq 0$	1
6.	y	$\frac{x}{y} = 7$	0
7.	x	$x^2 = 1$ and $y^2 = 9$	y
8.	ab	$a = \frac{2}{3}n$ and $b = \frac{3}{2}n$	n^2

ANSWER CHOICES

A if the quantity in Column A is the greater
B if the quantity in Column B is the greater
C if the two quantities are equal
D if the relationship cannot be determined from the information given

	Column A	Column B
9.	ABCD is a rhombus $m\angle A = 60^0$	perimeter ΔABC
10.	$x > y > 0$ $\frac{x^2 y^3}{xy^4}$	$\frac{x^3 y^2}{x^4 y}$
11.	$\left(\sqrt{3}\right)^2$	$\left(\sqrt{3} - 1\right)\left(\sqrt{3} + 1\right)$
12.	 18	x
13.	$a > b > c > 0$ $\frac{a}{b}$	$\frac{c}{a}$
14.	$x \neq 0$ $x(y - z)$	$-x(y + z)$
15.	a, b, c, d are positive and $\frac{a}{b} = \frac{c}{d}$ $\frac{a}{b}$	$\frac{a + c}{b + d}$
16.	$9x + 5y = 4$ x	y
17.	$kx > ky$ x	y
18.	 AB = AC $m\angle A$	$m\angle B$

	Column A	Column B



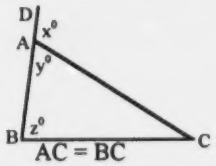
AB = BC

Each of the three arcs are semicircles.

19. length of APC | length of AQB + BRC

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

20. $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$ | $\begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix}$



AC = BC

21. x | y

---

n is a two-digit integer

22. 10n | n^2

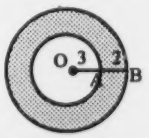
---

23. The measure of the acute angle formed by the hands of a clock at 2:00 | The measure of the acute angle formed by the hands of a clock at 4:30

$0 < r < 2$

24. The area of a circle with radius r | The circumference of a circle with radius r

---



OA = 3
AB = 2

The two circles are concentric at O

25. area of smaller circle | area of shaded region

---

m, n, and k are positive

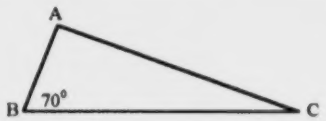
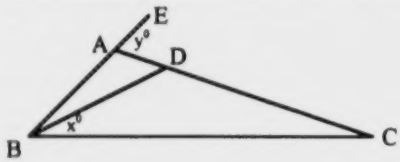
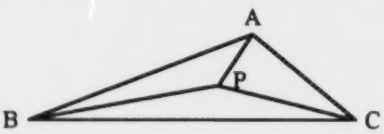
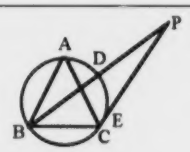
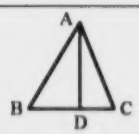
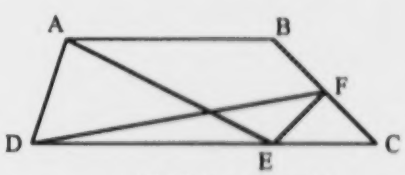
26. $\frac{m}{n}$ | $\frac{m + k}{n + k}$

---

27. $2 + \frac{1}{2 + \frac{1}{2}}$ | $3 + \frac{1}{3 + \frac{1}{3}}$

ANSWER CHOICES

A if the quantity in Column A is the greater
B if the quantity in Column B is the greater
C if the two quantities are equal
D if the relationship cannot be determined from the information given

	Column A		Column B
28.	$(AB)^2 + (BC)^2$		$(AC)^2$
29.	x		y
30.	AB + BC + AC		2 (AP + PB + PC)
31.	60^0	 ΔABC is equilateral	$m\angle P$
32.	AB		AD
33.	area ΔADE	 ABCD is a trapezoid	area ΔDFE

Student-Produced Response Questions

Questions of this type have no answer choices provided. Instead, you must solve the problem and fill in your answer in a special grid. Ten questions on the test will be of this type.

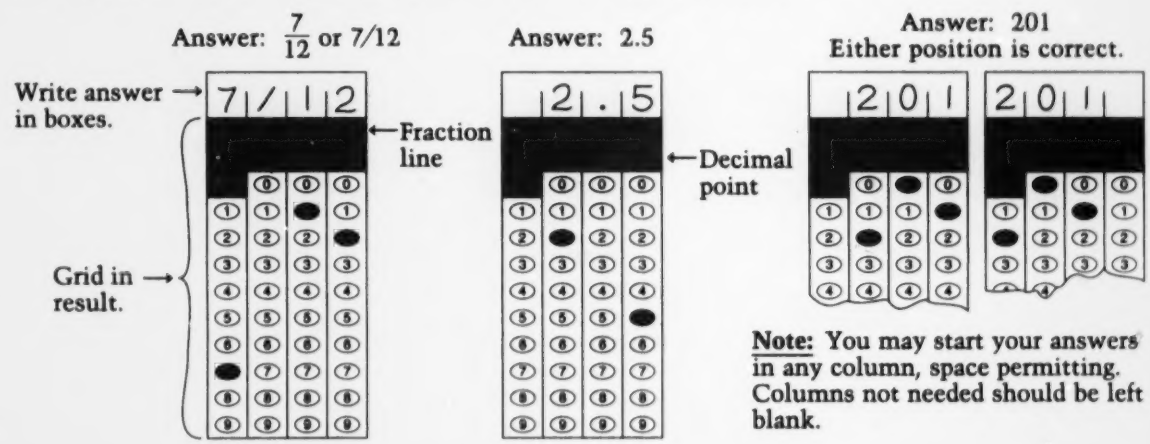
It is very important for you to understand the directions for entering answers on the grid! Valuable testing time will be lost if you read the directions for the first time when you take the test. The directions are fairly simple and the gridding technique is similar to the way other machine-readable information is entered on forms.

A primary advantage of this format is that it allows you to enter the form of the answer that you obtain, whether whole number, decimal, or fraction. For example, if you obtain $\frac{2}{5}$, you can grid 2/5. If you obtain .4, you can grid .4. Generally, you should grid the form of the answer that you obtain naturally in solving the problem. The grid will only hold numbers that range from 0 to 9999. Decimals and fractions can also be gridded.

Below are the actual directions that you will find on the test — read them carefully. Practice gridding your answers. At the end of the examples, you will find one way to grid the correct answer to each question.

Directions for Student-Produced Response Questions

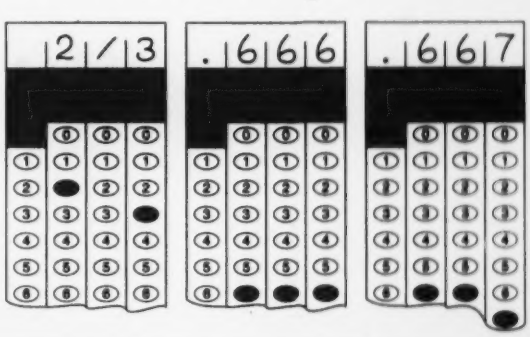
Each of the remaining 10 questions requires you to solve the problem and enter your answer by marking the ovals in the special grid, as shown in the examples below.



Note: You may start your answers in any column, space permitting. Columns not needed should be left blank.

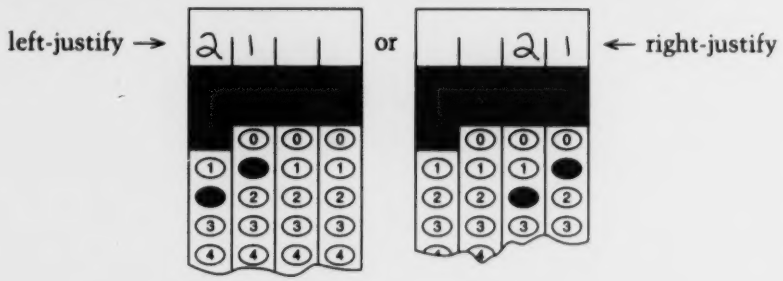
- Mark no more than one oval in any column.
- Because the answer sheet will be machine-scored, **you will receive credit only if the ovals are filled in correctly.**
- Although not required, it is suggested that you write your answer in the boxes at the top of the columns to help you fill in the ovals accurately.
- Some problems may have more than one correct answer. In such cases, grid only one answer.
- No question has a negative answer.
- **Mixed numbers** such as $2\frac{1}{2}$ must be gridded as 2.5 or 5/2. (If 2 1/2 is gridded, it will be interpreted as $\frac{21}{2}$, not $2\frac{1}{2}$.)
- **Decimal Accuracy:** If you obtain a decimal answer, **enter the most accurate value the grid will accommodate.** For example, if you obtain an answer such as 0.6666 . . . , you should record the result as .666 or .667. **Less accurate values such as .66 or .67 are not acceptable.**

  Acceptable ways to grid $\frac{2}{3}$ = .6666 . . .



Hints for Filling in Answer Grids

- Decide in which column of the grid you want to begin your answers. We recommend that you begin gridding in the first (left-hand) column of the grid or that you right-justify your answers (see below). You do not want to be thinking about where to begin gridding during the test, because you may waste valuable testing time.



- Answers can begin in any column of the grid as long as the entire answer will fit. For example, an answer of 7 can be entered in any of the four columns.
- If the answer is 0, it can be gridded in column 2, 3, or 4. Zero has been omitted from column 1 to encourage that the most accurate value be gridded for certain answers. For example, an answer of 1/8 could also be gridded as .125 but not as 0.12, which is less accurate.
- Unused columns in the grid should be left blank.
- If you obtain a fraction such as $\frac{9}{15}$, it does not have to be reduced. Fractions do not need to be reduced unless they will not fit the grid. For example, $\frac{15}{25}$ will not fit the grid; you would need to grid $\frac{3}{5}$ or .6 instead.
- Do your best to make sure your answer is correct before you grid the result. If you need to erase your answer, do so *completely* since incomplete erasures in a column may be picked up by the scoring machine as intended answers.
- If you obtain a decimal, enter the most accurate value the grid will accept. For example, if you obtain 1.333 . . . fill in all four spaces on the grid by entering 1.33, *not* 1.3.
- Mixed numbers such as $2\frac{1}{2}$ *cannot* be gridded in mixed number form (see directions).

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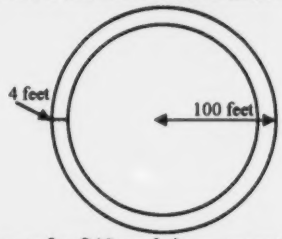
MATHEMATICS TEST 21
Time Limit: 30 Minutes
STUDENT-PRODUCED RESPONSE QUESTIONS

1. What fraction of the area of an $8\frac{1}{2}$ by 11 inch piece of paper remains if the length is reduced by $2\frac{1}{2}$ inches and the width is reduced by $1\frac{1}{2}$ inches?

2. A group of 5 workers has contracted to hoe a 30-acre field for $1,200. If it takes the group 4 days to complete the job, working 10 hours a day, what is the hourly wage each worker earns?

3. How many ounces of wine of 12% alcohol content contain an amount of alcohol equivalent to 4 ounces of whiskey with an alcohol content of 40%?

4. A groundskeeper mows a circular lawn, using a mower that takes out 4-foot bands, as shown. If the radius of the lawn measures 100 feet, what percent of the lawn has been mowed after 20 revolutions of the mower (with no mowing overlaps)?



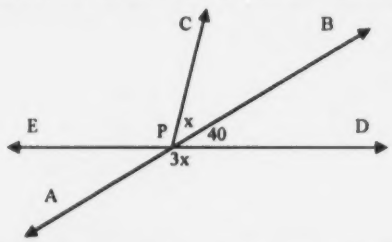
5. The revenue R brought in by selling N lamps for $40 each is represented by R = $40N. The cost of manufacturing N lamps is represented by C=$200 + $10N. What is the least number of lamps the manufacturer must sell to show a profit?

6. From 1981 to 1982, the population of a town grew by 3%; from 1982 to 1983, it increased by 6%. What was the percent of population growth for the period from 1981 to 1983?

7. At birth, a child's head measures about $\frac{1}{3}$ the length of her body. At age 1, the child's head is $\frac{1}{4}$ the length of her body. If a child's height measured 18 inches at birth and she grew 8 inches in the first year, what was the ratio of the growth of her head to the growth of her entire body?

8. A college issues student IDs stamped with a letter followed by 2 numbers. The letter O is not used, and no ID may contain one digit repeated, such as T55. How many ID numbers may be issued using this system?

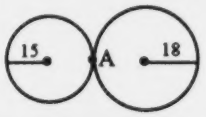
9. In the diagram below, $\overline{AB}$ intersects $\overline{DE}$ and $\overline{PC}$ at point P. If $\angle CPB = x$, $\angle APD = 3x$ and $\angle BPD = 40$, find the measure of $\angle CPE$.



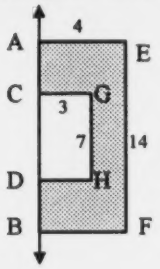
10. Joyce is building a shelf with 8-inch-wide cinder blocks and a 6-foot-long board. She spaces the cinder blocks apart from each other equally so the board extends 6 inches beyond each block. If she uses 4 cinder blocks to hold up the board, how many inches lie between each pair of adjacent blocks?

11. You are visiting a friend who lives $1\frac{1}{2}$ miles east and 1 mile north of you. After walking $\frac{3}{4}$ mile east, you are able to take a "diagonal route" directly to your destination. What is the total distance you have walked?

12. Two gears interact as shown. Their radii measure 15 and 18 inches, respectively. What is the least number of revolutions of the larger gear, so that both gears are at the starting point A of each wheel?



13. Find the volume that the shaded region generates when it makes one revolution on axis $\overline{AB}$. AEFB and CDHG are rectangles, whose dimensions are 4, 14 and 3, 7, respectively. (use $\pi = \frac{22}{7}$).



14. A palindrome is a number that reads the same in both directions (e.g. 12,321). How many palindromes are there between 100 and 1000?

15. Carol and Grace leave the shore, swimming out to a buoy 900 meters away, and return. Grace is swimming at $\frac{2}{3}$ Carol's rate. If Carol's total time for the round trip is 16 minutes, how many meters will Grace have swum when Carol meets her on return?

MATHEMATICS TEST 22
Time Limit: 30 Minutes
STUDENT-PRODUCED RESPONSE QUESTIONS

1. If $3^{3x+1} = 27^{2x}$, what is the value of x?

2. Find the slope of a line which goes through the origin and the point (-3,-4).

3. The operation * is defined as $a * b = a^b + b$. Find $\frac{1}{2} * 2$.

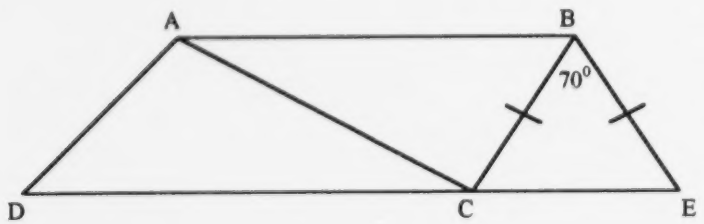
4. How many hours will it take two cars, starting at the same time and traveling toward each other at 40 mph and 60 mph, respectively, to meet, if their starting points are 200 miles apart?

5. Suppose one newspaper comes out every 3 days and another every 5 days. Assuming both come out on New Year's Day, how many days of the year will the two newspaper come out together?

6. Find the value of $3^{99} + (-3)^{99}$.

7. A bushel is a unit of volume measuring approximately 1.25 cubic feet. How many full bushels of grain can a 30-foot-high cylindrical silo with a 10-foot radius hold? (Use $\pi = \frac{22}{7}$).

8. ABCD is a parallelogram with diagonal AC. ΔBCE is an isosceles triangle and $m\angle CBE = 70^0$.
Find the measure of $\angle DAB$.



9. 20 teams are playing in a single elimination softball tournament (a team drops out after one loss). How many games are played until a winner is declared?

10. The circle graphed on the x,y-plane has center $(1,\sqrt{3})$. If it is rotated counterclockwise 30^0 about the origin, what will be the value of the y-coordinate of the new center?

11. On average, motorcycles are involved in accidents 4 times as often as automobiles and 8 times as often as trucks. If in 1995 there were 55 vehicle accidents in one town, at this rate, how many involved automobiles?

12. David participates in a bicycle race in which a rider starts every 5 minutes. David's average rate is 33 km/hr. After 28 minutes, he is overtaken by a rider who started 2 persons after him. What is the rate of the other cyclist in km/hr?

13. Suppose in a given year that the average life expectancy for males in the U.S. is 67 years for an 18-year-old, 68 years for a 30-year-old, 70 years for a 42-year-old, and 73 years for a 54-year-old. Assuming this trend continues, find the average life expectancy for a 78-year-old male.

14. To conserve the contents of a 16 oz bottle of wine, an alcoholic adopts the following procedure. On the first day, he drinks 1 oz of wine and then refills the bottle with water. On the second day, he drinks 2 oz of the mixture and then refills the bottle with water. On the third day, he drinks 3 oz of the mixture and again refills the bottle with water. The procedure is continued for succeeding days until the bottle is empty. How many ounces of water does he drink?

15. If the distance between the circumferences of two concentric circles is 7, what is the difference between their circumferences? (Use $\pi = \frac{22}{7}$).

V. Solutions for Practice Tests

(An answer key for rapid checking of your results will be found at the end of the book.)

The following solutions to the problems in the practice tests of Part IV will help you discover the reasons for your wrong answers and will show you how to solve the problems you could not do at all. In many instances, more than one method of solution is shown. Therefore, even your correct answers should be checked by comparing your method with these solutions, to learn whether there is a simpler or faster way of working the problem than the one you used. This part of the book will be of little value, however, unless you make a serious attempt to answer all the questions before consulting the solutions. On the other hand, doing the tests and then studying the solutions can have a significant effect on your score on the mathematics section of the SAT-I.

In this section the symbol $\angle A$ represents both the angle and its measure, the symbol AB represents both the line segment and its measure, and the symbol $\overset{\frown}{AB}$ represents both the arc and its measure.

TEST 1 SOLUTIONS

1. (B) If $\frac{1}{y} = \frac{3}{5}$, then by inverting both sides, $\frac{y}{1} = \frac{5}{3}$, or $y = \frac{5}{3}$

Alternate Solution: If $\frac{1}{y} = \frac{3}{5}$, then by cross multiplying we get $3y = 5$, and $y = \frac{5}{3}$.

2. (D) By adding the fractions in the numerator and in the denominator, we get $\frac{8\left(\frac{1}{3}\right)}{8\left(\frac{1}{5}\right)} = \frac{\frac{1}{3}}{\frac{1}{5}}$.

There are two possible methods of simplification:

a. $\frac{\frac{1}{3}}{\frac{1}{5}} \cdot \frac{15}{15} = \frac{5}{3} = 1\frac{2}{3}$

b. $\frac{1}{3} \div \frac{1}{5} = \frac{1}{3} \cdot \frac{5}{1} = \frac{5}{3} = 1\frac{2}{3}$

3. (E) Substitute $5y - 2$ for x in $3x - 4$. Then $3(5y - 2) - 4 = 15y - 6 - 4 = 15y - 10$

4. (E) Since all the choices are in terms of x, replace z by x:

$x^2 y = 1$ and $y = \frac{1}{x^2}$

5. (A) The only way in which $\frac{n}{5}$ may be greater than n is if n is negative. Therefore, if $n = -20$ then $\frac{-20}{5} = -4$, which is greater than -20.

6. (A) If a is replaced by b, then both sides of the equation would be identical. Therefore, I is true.

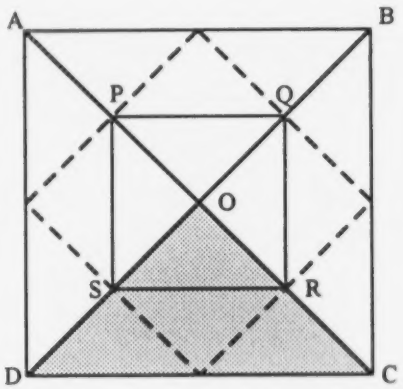
If a is replaced by $\frac{1}{b}$ (choice IV), then $a + \frac{1}{a} = \frac{1}{b} + \frac{1}{\frac{1}{b}} = \frac{1}{b} + b$, again making both sides of the equation equal. Therefore, IV is true. Similarly, it may be shown that II and III are not true.

7. (C) Carol pays 25¢ for the first mile. After she has traveled one mile, she must still travel $(p - 1)$ miles at the rate of 5¢ per mile. For these $(p - 1)$ miles she must pay $[5(p - 1)]$¢. The total fare = $(25 + 5p - 5)$¢ = $(20 + 5p)$¢.

8. (C) Here we must assume that Barbara is very unlucky: On her first four draws, she is unable to get a pair of jackets of the same color. However, since she only has jackets of four different colors, she must match one of her previous draws on her fifth draw. Therefore, she is assured of getting a pair of jackets of the same color in five draws.

9. (D) Since the corresponding linear parts (i.e., sides, diagonals, etc.) of squares PQRS and ABCD are in the ratio $\frac{1}{2}$, their areas are in the ratio $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$.

Alternate Solution 1:



By drawing the dotted lines as shown in the adjoining diagram, it may be seen that the area of ΔSOR is $\frac{1}{4}$ the area of ΔDOC, which is $\frac{1}{4}$ the area of square ABCD. Therefore, the area of ΔSOR is $\frac{1}{16}$ the area of square ABCD. However, the area of square PQRS is equal to 4 times the area of ΔSOR; hence the area of square PQRS is $\frac{4}{16}$ or $\frac{1}{4}$ the area of square ABCD.

Alternate Solution 2: Since the area of square PQRS $= \frac{1}{2}(PR)^2$ and $PR = \frac{1}{2}(AC)$,

the area of square PQRS $= \frac{1}{2}\left[\frac{1}{2}(AC)\right]^2 = \frac{1}{2}\left[\frac{1}{4}(AC)^2\right]$. However, the area of square ABCD $= \frac{1}{2}(AC)^2$.

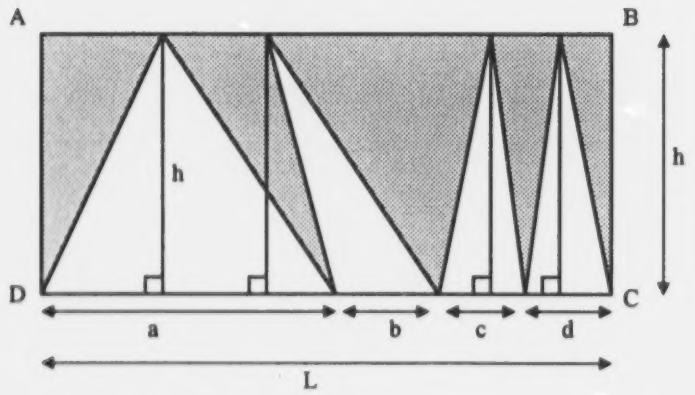
Thus the area of square PQRS $= \frac{1}{4}$ the area of square ABCD.

10. (D) Let x = the fourth number. We may then represent the average of the four numbers as $\frac{3 + 5 + 9 + x}{4} = 8$. By solving the equation $\frac{17 + x}{4} = 8$, we get $x = 15$.

11. (D) Let x = the number of seconds in which the record revolves 180^0, or $\frac{1}{2}$ revolution. Then

$\frac{x \text{ sec.}}{60 \text{ sec.}} = \frac{\frac{1}{2} \text{ revolution}}{33\frac{1}{3} \text{ revolutions}}$. By cross multiplying: $\frac{100x}{3} = 30$ and $x = \frac{9}{10}$ sec.

12. (C)



Consider the sum (S) of the areas of the nonshaded triangles (all having altitude h).

$$S = \frac{1}{2}ah + \frac{1}{2}bh + \frac{1}{2}ch + \frac{1}{2}dh$$

$$S = \frac{1}{2}h(a + b + c + d).$$

$a + b + c + d = L$, the base of the rectangle.

Therefore, $S = \frac{1}{2}$ hL. However, the area of rectangle ABCD =hL. If the area of the nonshaded region is one half the area of the rectangle, then the area of the shaded region must also be one half the area of the rectangle. Hence the required ratio is $\frac{1}{2}$.

13. (A) This series of numbers adheres to the following pattern: Starting with 1, we must multiply by 3 to get the next term, then subtract 1 (from this result) to get the term following that. A continuation of this process will generate the given series:

1 x 3 = 3	15 - 1 = 14
3 - 1 = 2	14 x 3 = 42
2 x 3 = 6	42 - 1 = 41
6 - 1 = 5	41 x 3 = 123
5 x 3 = 15	123 - 1 = 122

14. (B) Since ΔPQT is a right triangle, we may apply the Pythagorean Theorem: $8^2 + (QT)^2 = 17^2$; $QT = 15$ (remember: This is a Pythagorean triple, 8, 15, 17). In a similar manner, we find RT: $9^2 + (RT)^2 = 15^2$; $RT = 12$ (remember: The sides of this right triangle are multiples of 3, 4, 5). Again applying the Pythagorean Theorem, this time to ΔRST: $(ST)^2 = \left(3\frac{1}{2}\right)^2 + 12^2$ $ST = 12\frac{1}{2}$.

(Remember: the sides of this right triangle are multiples of 7, 24, 25).

15. (A) Since a krone is worth 50 francs and a pound is worth $17\frac{1}{2}$ krone, a pound is worth $(17\frac{1}{2})$ (50) or 875 francs. Since a dollar is worth 350 francs, a pound is worth $2\frac{1}{2}$ or $\frac{5}{2}$ dollars. One dollar is therefore equal to $\frac{2}{5}$ or .4 pounds.

16. (B) If the department reduces from the present 2000 employees to 1500 employees, the department is now $\frac{3}{4}$ of its original size. If Division C is to be reduced proportionately with the others, we must retain only $\frac{3}{4}$ of the original 600 employees, or 450 employees.

17. (C) Let x = the number of hours it would take Richie to do the whole job. Then $\frac{\frac{2}{5}\text{ job}}{1\text{ job (whole)}} = \frac{\frac{1}{5}\text{ hour}}{\text{x hour}}$.

By cross multiplying: $\frac{2}{5}x = \frac{1}{5}$, and $x = \frac{1}{2}$

18 (C) The fraction $\frac{\text{sum of area of circles}}{\text{area of rectangle}}$ represents the part of the rectangle which was cut out. To change this fraction to a percent, multiply by 100%. Since each circle has an area of $\frac{1}{4}\pi$, the sum of the areas of the four circles is π. Hence our desired fraction is $\frac{\pi}{24}$, which when changed to percentage form is $\frac{100\pi}{24}\%$ or $\frac{25\pi}{6}\%$.

19. (E) A job requiring 2 people, 2 hours a day for 2 days may be said to be (2)(2)(2) "man-hours." Similarly, the size of the second job would be(6)(6)(6) "man-hours." Let x represent the number of articles produced in the second job. We set up the proportion: $\frac{2\text{ articles}}{x\text{ articles}} = \frac{(2)(2)(2)\text{ man-hours}}{(6)(6)(6)\text{ man-hours}}$; hence $x = 54$.

20. (B) After $\frac{2}{3}$ of the votes, $\frac{2}{3}V$, have been counted, Edith has $\frac{5}{6}$ of what she needs to win, $\frac{3}{4}V$. Therefore, she now has $\frac{5}{6}(\frac{3}{4}V)$ or $\frac{5}{8}V$. Since she needs $\frac{3}{4}V$ to win, she still needs $\frac{1}{8}V$. However, $\frac{1}{3}$ of the votes have not yet been counted. Hence $\frac{\frac{1}{8}V}{\frac{1}{3}V}$ represents the part of the remaining votes which Edith needs. Simplifying, we find that Edith needs $\frac{3}{8}$ of the remaining votes.

TEST 2 SOLUTIONS

1. (B) Let us consider $\frac{11}{32}$.

[Choice (A)]: $\frac{11}{32} > \frac{8}{32} = \frac{1}{4}$, and $\frac{11}{32} > \frac{11}{33} = \frac{1}{3}$.

Therefore, $\frac{11}{32}$ is not in the interval from $\frac{1}{4}$ to $\frac{1}{3}$. Let us now consider the second fraction $\frac{4}{13}$.

[Choice (B)]: $\frac{4}{13} > \frac{4}{16} = \frac{1}{4}$ and $\frac{4}{13} < \frac{4}{12} = \frac{1}{3}$.

Therefore, $\frac{4}{13}$ is in the interval from $\frac{1}{4}$ to $\frac{1}{3}$.

Choice (C) lies outside the interval, since $\frac{3}{16} < \frac{4}{16} = \frac{1}{4}$.

Choice (D) lies outside the interval, since $\frac{6}{17} > \frac{6}{18} = \frac{1}{3}$.

Choice (E) lies outside the interval, since $\frac{6}{25} < \frac{6}{24} = \frac{1}{4}$.

2. (C) The only expression which increases as x increases is x - 1, since you are always subtracting a constant from an increasing term.

3. (A) Let R be the radius of this circle. Then $\pi R^2 = 2 \pi R$, and $R = 2$

4. (B) If $\frac{x}{y} = 1$, then $x = y$. If $x = y$ and $x + y = 1$, then $x = y = \frac{1}{2}$. Hence $xy = \frac{1}{2} \bullet \frac{1}{2} = \frac{1}{4}$.

5. (B) Consider the differences between consecutive terms: 7, 14, 21, 28. The next difference must be 35. Therefore, by adding 35 to the last number (72) in the original series, we get the next number, 107.

6. (E) This can probably be answered as quickly without a calculator as with one. Since (A), (B), and (D) are all very close to one another in size, let us first inspect (C) and (E) to see if they are possible answers.

(C) $\frac{9}{\frac{10}{9}} = 9 + \frac{10}{9} = 9 \bullet \frac{9}{10} = \frac{81}{10} = 8\frac{1}{10} = 8.1$, which is larger than (A), (B) or (D).

(E) $\left(\frac{9}{10}\right)^2 = \frac{81}{100} = .81$, which is clearly less than the other choices.

Note: A bit of preliminary inspection of the choices enabled some valuable savings of time.

7. (A) Since the opposite angles of an inscribed quadrilateral are supplementary (i.e., have a sum of 180^0), $x = 120^0$.

8. (D) If we remove 1: $5 + 7 = 3 + 9$, which violates the second condition. If we remove 5: $3 + 7 = 1 + 9$, which again violates the second condition. If we remove 7: $1 + 3 + 5 = 9$, which violates the first condition. If we remove 9: $1 + 7 = 3 + 5$, which violates the second condition.

If we remove 3: $1 \neq 5 + 7 + 9$ $\quad 1 + 5 \neq 7 + 9$

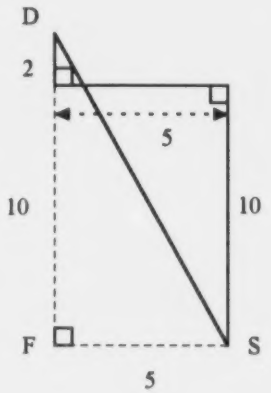
$5 \neq 1 + 7 + 9$ $\quad 1 + 7 \neq 5 + 9$

$7 \neq 1 + 5 + 9$ $\quad 1 + 9 \neq 5 + 7$

$9 \neq 1 + 5 + 7$

Hence both conditions are satisfied when 3 is removed from the set

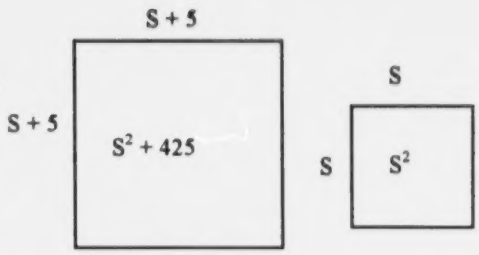
9. (C) In order to find the distance from S to D draw the dotted lines as indicated in the adjoining diagram. Since SF = 5, and FD = 10 + 2 = 12, DS = 13, by the Pythagorean theorem (ΔDFS).



10. (B) The circumference of the wheel is 10π inches. Therefore, the fixed point travels 300 (10π) inches in 1 minute. We set up the following proportion $\frac{300(10\pi) \text{ inches}}{x \text{ inches}} = \frac{60 \text{ second}}{1 \text{ second}}$, where x is the number of inches the point travels in one second. Then $x = 50\pi$.

11. (A)

Let S = the side of the original square. The area of the new square is $s^2 + 425$. Since each side of the new square is S + 5, the area of the new square is $(S + 5)^2$.
Therefore $(S + 5)^2 = S^2 + 425$.
$S^2 + 10S + 25 = S^2 + 425$, and S = 40.



12. (B) If P is on the circle, then $m\angle BPC = 60^0$, because it would be measured by $\frac{1}{2}\overset{\frown}{BC}$. If P is inside the circle, then $m\angle BPC > 60^0$. Therefore, since P is outside the circle, $m\angle BPC < 60^0$. Also note that $\angle BPC$, formed by two secants, is measured by one half a quantity less than 120^0, or less than 60^0 (i.e. $\overset{\frown}{BC}$ - another arc).

13. (E) We must compare the cross-section of each of the pipes: Since the ratio of the diameters is $\frac{3}{2}$, the ratio of the areas of the cross-section is $(\frac{3}{2})^2$ or $\frac{9}{4}$. Since the area of the cross-section of each pipe varies inversely as the time it requires to empty the tank, we set up the following proportion: $\frac{9}{4} = \frac{x \text{ minutes}}{4 \text{ minutes}}$; therefore, x = 9.

14. (E) Let b = the weight of the bottle
Let t = the weight of the tumbler
Let p = the weight of the pitcher
Let *l* = the weight of the plate
b + t = p; (1)
b = *l* + t, or b - t = *l*; (2)
2p = 3*l* (3)
Substituting in equation (3):
2(b + t) = 3(b - t) and b = 5t

15. (D) This problem may be written as 5x = 6y, where x and y represent the number of chimes of each. The smallest positive integral solution is x = 6, y = 5. Hence the bells will chime simultaneously in 30 seconds.

16. (B) Since five widths are equal to four lengths, $\frac{l}{w} = \frac{5}{4}$. The area of each small rectangle = *l*w = 5.

Since $l = \frac{5w}{4}$, $\left(\frac{5w}{4}\right)$ w = 5, and w = 2.

Therefore $l = \frac{5}{2}$. The perimeter of the large rectangle = $2(5w + w + l) = 2[5(2) + 2 + \frac{5}{2}] = 29$.

17. (E) The volume of the first can is $4\left(\frac{5}{4}\right)^2\pi$, and the volume of the second can is $5\left(\frac{3}{2}\right)^2\pi$. ($V = \pi R^2 h$)

We set up the following proportion: $\frac{4 \bullet \frac{25}{16} \bullet \pi}{5 \bullet \frac{9}{4} \bullet \pi} = \frac{10}{x}$, where x, = number of ounces contained in the second can. Thus x = 18.

18. (E)

$$\frac{1}{4} + 4\left(\frac{1}{12} + \frac{1}{x}\right) = \frac{1}{4} + 4\left(\frac{x + 12}{12x}\right) = 1\frac{3}{4}$$

$$4\left(\frac{x + 12}{12x}\right) = 1\frac{1}{2} = \frac{3}{2}$$

$$\frac{x + 12}{12x} = \frac{3}{8}$$

$$48\left(\frac{x + 12}{12x}\right) = 48\left(\frac{3}{8}\right) = 18$$

Therefore: $3 + 48\left(\frac{x + 12}{12x}\right) = 18 + 3 = 21$, or 12 times the size of the original expression.

19. (E) In city A, 25 units were built for every 10,000 people, and there were 250,000 (25 x 10,000) people in the city. Hence(25)(25) = 625 new dwelling units were built in city A.

20. (A) After Ernie works for 20 hours, he completes $\frac{20}{36} = \frac{5}{9}$ of the job. Since the remaining $\frac{4}{9}$ of the job is done by both (working together) in 10 hours, we get $\frac{4}{9} = \frac{10}{x}$, where x is the number of hours in which both (working together) can complete the entire job. Therefore, $x = 22\frac{1}{2}$.

TEST 3 SOLUTIONS

1. (E) Choices (A) and (D) say the same thing in a different format. Therefore, neither can be the answer. Choices (C) and (E) have a "similar appearance" but seem to contradict each other. Therefore, one of them must be wrong. The wrong one is (E), which is the answer to the question.

2. (E) If there are 2(36) inches in 2 yards, then there are 36y inches in y yards.

3. (B) In the right triangle: $78^0 + x = 90^0$. Therefore, $x = 12^0$ and $5x = 60^0$. In the isosceles triangle: $2y + 60^0 = 180^0$, and $y = 60^0$.

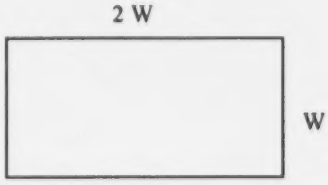
4. (B) If Evelyn has her wages cut by $\frac{1}{5}$, then she receives $\frac{4}{5}$ of her original wages. To regain her original wages, she must receive the $\frac{1}{5}$ she lost. This $\frac{1}{5}$ is $\frac{1}{4}$ of the $\frac{4}{5}$ she has, or 25%.

5. (D)

Let w = the width, then 2w = the length,
x = the perimeter.

$2(w + 2w) = x$, $6w = x$,

$w = \frac{x}{6}$ (width), $2w = \frac{x}{3}$ (length).



Therefore the area $= \left(\frac{x}{6}\right)\left(\frac{x}{3}\right) = \frac{x^2}{18}$.

6. (C) Since rate × time = distance, and 72 minutes = $\frac{6}{5}$ hours:

$R\left(\frac{6}{5}\right) = \frac{2}{5}$; $R = \frac{1}{3}$ miles per hour.

7. (D) $\dfrac{\frac{1}{x} - \frac{1}{y}}{\frac{1}{xy}} \cdot \frac{xy}{xy} = \frac{y - x}{1} = y - x$

8. (A) The size of the job is (12) (5) "man-days". Let x = the number of days in which 8 carpenters can do the job. Therefore, (12)(5) = 8x; $x = 7\frac{1}{2}$.

9. (E) The general term, the n^{th} term, is $n^2 - n$. That is, the 4^{th} term is $4^2 - 4 = 12$. Hence, the 7^{th} term (the missing term) is $7^2 - 7 = 42$.

Alternate Solution:
Since the differences between the terms increase by 2 with each new term, the difference between 30 and the next term must be 12; hence the next term must be 42.

10. (B) Writing the percents as fractions: $x = \frac{1}{4}y$ and $y = \frac{4}{5}z$. Substituting for y in the first equation yields:

$x = \frac{1}{4}\left(\frac{4}{5}z\right) = \frac{1}{5}z$ or 20% of z.

11. (E) If $\angle BPA = \angle DPC = 70^0$, then in ΔAPB: $\angle BAP + \angle APB + \angle ABP = 180^0$, and $\angle BAP + 70^0 + 20^0 = 180^0$. Since $\angle BAP = 90^0$, $\angle BDC = 90^0$. (Since both are measured by one half arc BC).

12. (D) Since the ratio of boys to girls is $\frac{5}{3}$, the average grade of the class may be found by assuming that there are 5 boys and 3 girls in the class, in which case the average is $\dfrac{5\,(80) + 3\,(72)}{5 + 3} = 77$.

13. (C) Multiplying Susan's second purchase by 3, we see that 1 yard of red, $1\frac{1}{2}$ yards of blue, and 1 yard of white cost \$9.00. We know from the first purchase that 1 yard each of all three colors costs \$7.50. Therefore, the extra $\frac{1}{2}$ yard of blue costs \$1.50 and 1 yard of blue cloth costs 2 (\$1.50) = \$3.00.
Symbolically:

$$3\left[\frac{1}{3}r + \frac{1}{2}b + \frac{1}{3}w\right] = 3\,(3.00)$$

$$r + 1\frac{1}{2}b + w = 9.00 \qquad (1)$$

$$r + b + w = 7.50 \qquad (2)$$

Subtracting (2) from (1) $\frac{1}{2}b = 1.50$

$b = 3.00$

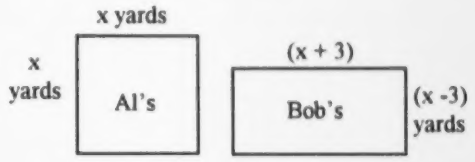
14. (D) Consider the following diagram : The numbers in each beaker represent the amount of liquid in them, whereas the number below the beaker represents the percent of alcohol in each. The following equation relates the *amount of alcohol* in each beaker: $.50(9) + .00(x) = .30(9 + x)$; $x = 6$.

15. (A) The job of stuffing 700 envelopes is (1) (1) "man days." If x represents the number of clerks that can stuff 14,000 envelopes in 5 days, then the second job is (5) (x) man-days.

Therefore: $\frac{700}{14{,}000} = \frac{(1)(1)}{(5)(x)}$, and $x = 4$.

16. (A) Let 4x yds. = the amount of fencing that Al and Bob each have. Since the length of Bob's rectangle is (x + 3) yd., the width of his rectangle must be (x - 3) yd.

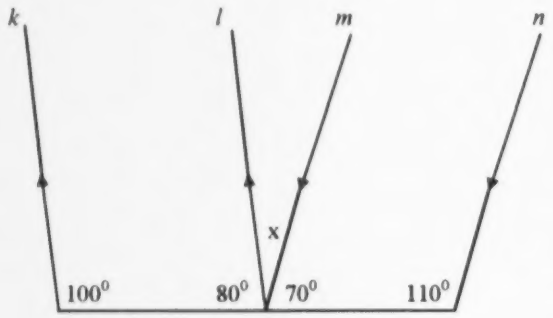
Al's area = x^2 sq. yd.;
Bob's area = $(x + 3)(x - 3) = (x^2 - 9)$ sq. yd.
Therefore, Al's area is larger than Bob's by 9 square yards.



17. (A) Let x represent the number of beds for infectious diseases, $1\frac{1}{2}$ x the number of beds for children's diseases, and 4x the number of beds for general ailments. Therefore, $x + 1\frac{1}{2}x + 4x = 650$; $x = 100$.

However, the number of beds for children's diseases is $1\frac{1}{2}x = 150$.

18. (C)



(80^0 and 70^0 are supplementary to 100^0 and 110^0 respectively) $80^0 + x + 70^0 = 180^0$; $x = 30^0$

19. (C) Let x = the amount of the first mixture taken to produce the desired mixture (18 oz.), then 18 - x = the amount of the second mixture used. $\frac{3}{8}x$ = the amount of first medicine in first mixture. $\frac{3}{10}(18 - x)$ = the amount of first medicine in second mixture. By adding the amount of first type of medicine in each beaker:
$\frac{3}{8}(x) + \frac{3}{10}(18 - x) = 6$; $x = 8$.

20. (C) When two tangents are drawn to the same circle from an external point, they are equal. Therefore: DB = BF, CE = CF; AD = AE = 20. The perimeter of ΔABC = AB + BF + FC + AC = (AB + BD) + (EC + AC) = AD + AE = 20 + 20 = 40.

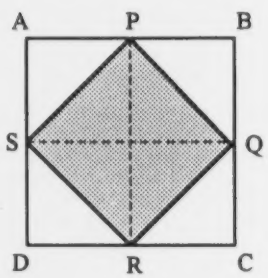
TEST 4 SOLUTIONS

1. (C) Find what part x is of y: $\frac{x}{y}$; multiply by 100 to change to percent form: $\frac{100x}{y}$.

2. (C) Add the given equations:

$$\begin{array}{rl} & 4a - 3b = 7 \\ & 3a + 2b = 19 \quad \text{Multiply by 2: } 14a - 2b = 52 \\ \text{Therefore: } & 7a - b = 26 \end{array}$$

3. (B) Draw dotted lines as shown in the adjoining diagram. Since each quarter of the shaded square is one half each quarter of square ABCD, the area of square PQRS = $\frac{1}{2}$ the area of square ABCD = $\frac{1}{2}(4) = 2$.



4. (D) If Max spends $\frac{1}{3} + \frac{1}{4} + \frac{1}{6} = \frac{3}{4}$,
then he still has $1 - \frac{3}{4} = \frac{1}{4}$ or 25% of his money left.

5. (D) If the sum of three consecutive integers is 45, then the middle integer (the average) is 15. Therefore, the next three consecutive integers are 17, 18, 19 and their sum = 3 (18) = 54.

6. (C) To find the area between the two concentric circles, subtract the area of the smaller circle from that of the larger circle; that is, $36\pi - 25\pi = 11\pi$.

7. (D) We may restate the question as: "How many '6 ounces' are contained in (12)(32) ounces?"
That is, $\frac{(12)(32)}{6} = 64$.

8. (E) At 2:15, the minute hand is exactly on the "3." At the same time, the hour hand has traveled $\frac{1}{4}$ the distance through the 30^0 interval between "2" and "3." Therefore, the angle is $\frac{3}{4}$ of the interval ("2" - "3")
or $\frac{3}{4}(30^0) = 22\frac{1}{2}°$

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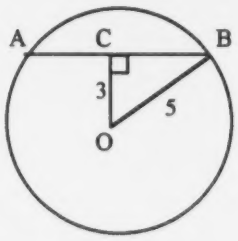
9. (C) Let *l* and w be the original length and width, respectively. Then (1.10) *l* and (1.10) w are the dimensions of the rectangle, whose area = (1.1) (*l*) (1.1) w = (1.21) *l*w, which represents an increase of 21% in area (original area = 1.00*l*w).

10. (A) In the adjoining diagram, side CB of right ΔBCO is 4 (by the Pythagorean Theorem). Therefore AB = 2 (4) = 8.



11. (B) The smallest number of committees are formed by having the greatest membership in each committee. Hence the solution is: two committees of 8 children each and two of 7 children each (i.e., four committees).

12. (E) If x - 2 is the smallest odd integer, then the next two consecutive odd integers are x and x + 2, respectively. The sum of these integers is 3x = 381, and x = 127, x - 2 = 125, and x + 2 = 129 are the integers.

13. (C) Since the diameter of the circle is $\frac{3}{16}$ inch, the radius is $\frac{3}{16} \cdot \frac{1}{2} = \frac{3}{32}$ inch. However, $\frac{3}{2}$ inch = 400 miles, and $\frac{3}{32} = \frac{1}{16} \cdot \frac{3}{2}$ inches = $\frac{1}{16} \cdot 400$ miles = 25 miles.
Therefore, the area of a circle whose radius is 25 miles is 625π square miles, or approximately 1960 square miles.

14. (D) To capture the first stone, the runner must travel 10 feet and return 10 feet. To capture the second stone, he must travel 15 feet and return 15 feet. This running plan implies the following sum: 2(10) + 2 (15) + 2(20) + 2(25) + 2 (30) = 20 + 30 + 40 + 50 + 60 = 200 feet.

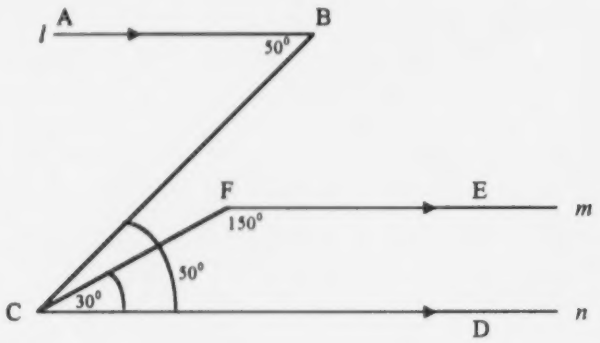
15. (C) The perimeter of the region is composed of two $\frac{2}{3}$-circumferences: triangles PQR and POR are equilateral, making $\angle ORQ = \angle OPQ = 120^0$, and arc ORQ = arc OPQ = 120^0. Hence, we seek $\frac{4}{3}$ of the circumference of one of the circles: $\frac{4}{3}(2\pi R) = 8\pi$.

16. (C) The sum of the positive variations = 0.7.
The sum of the negative variations = -1.6.
Therefore, the sum of all the variations = -.9.
If the average of the 6 variations = $\frac{-.9}{6}$ = -.15, then the reading of the instrument = 24.00 - .15 = 23.85.

17. (D) When the circumference of a circle is 20", the radius is $\frac{20"}{2\pi}$; when the circumference of a circle is 25", the radius is $\frac{25"}{2\pi}$. This change in radius is $\frac{25"}{2\pi} - \frac{20"}{2\pi} = \frac{5"}{2\pi}$.

18. (B) Since the ratio of the corresponding linear parts (diameters) of the three semicircles is 2:3:5, the ratio of their areas is $2^2:3^2:5^2$ or 4:9:25. If we assume that the largest semicircle has an area of 25, then the other two semicircles have areas of 4 and 9; and the area of the shaded region has an area of 12. Therefore, the ratio of the area of the shaded region to the area of the largest semicircle is 12:25.

19. (C)
If $\angle BCD = \angle ABC = 50^0$ (alternate interior angles) and $\angle DCF + \angle CFE = 180^0$, (consecutive interior angles) then $\angle FCD = 30^0$.
Therefore, $\angle BCF = \angle BCD - \angle FCD = 50^0 - 30^0 = 20^0$.



20. (B) Let C = the circumference of the circle

$\frac{2 \text{ hours}}{24 \text{ hours}} = \frac{\frac{1"}{2}}{C}$; $C = 6"$. Since $6 = 2\pi R$, $R = \frac{3}{\pi}$.

TEST 5 SOLUTIONS

1. (C) $-3 - (-7) = -3 + 7 = 4$

2. (E) When dividing by 3, the only possible remainders are 0, 1, 2. Therefore, choices (C) and (D) are wrong answers Since the remaining choices, (A), (B), and (E), have the same first number, we need only inspect the second number. Since 14 + 3 leaves a remainder of 2, the correct choice must be (E).

3. (B) Let x = number of instruments produced in 1 hour. We may set up the following proportion:

$$\frac{\text{n instruments}}{\text{x instruments}} = \frac{\text{h hours}}{\text{1 hour}}$$ Solving the equation for x yields $x = \frac{n}{h}$.

4. (E) Since *all the numbers are evenly spaced*, we may get the average of the 10 numbers by averaging the first and last: $\frac{20 + 29}{2} = 24\frac{1}{2}$. Other methods using a calculator are also available.

5. (D) Let x = the number to be determined. $\frac{1}{2}x - \frac{1}{7}x = 5$, and $x = 14$.

6. (E) Increasing the circumference by 20% increases the radius by 20%. If the ratio of the old radius to the new radius is 1:1.2, then the ratio of the old area to the new area is the square of the previous ratio, or 1:1.44. The new circle is greater than the original area by .44 or 44%.

7. (E) We seek $\left(\frac{a}{c}\right)$ 100%. $b = 83\frac{1}{3}\%\, c = \frac{5}{6}c$; $a = 60\%\, b = \frac{3}{5}b$. Since $a = \frac{3}{5}b$, $b = \frac{5}{3}a$.

Substituting in the first equation: $\frac{5}{3}a = \frac{5}{6}c$; $\frac{a}{c} = \frac{1}{2}$. Hence, $\left(\frac{a}{c}\right)$ 100% = 50%, and a is 50% of c.

8. (E) If x = the number of gallons the tank can hold, then $\frac{2}{3}x + 10 = \frac{5}{6}x$, and $x = 60$.

9. (D) If x = the number of miles Fred is *going*, then x must also equal the number of miles returning, $\frac{x}{10}$ is his time going, and $\frac{x}{4}$ is his time returning. Therefore $\frac{x}{10} + \frac{x}{4} = 5$, and $x = 14\frac{2}{7}$.

10. (B) $\frac{\text{Shaded squares}}{\text{Total squares}} = \frac{17}{81} \approx 21\%$.

11. (C) $\angle 1 = 45^0$, and ΔAOC is isosceles since radius AO = radius CO. Therefore $\angle A = \angle C$.
Hence $2x + 45^0 = 180^0$, and $x = 67\frac{1^0}{2}$.

12. (D) If 123 + 456 + 789 = 1368 is the sum of the first three numbers, then 2367 - 1368 = 999 is the sum of the next three numbers. Hence the average of these last three numbers is $\frac{999}{3} = 333$.

13. (B) The first person shakes hands with 5 others. The second person shakes hands with 4 others, all but the first person. The third person shakes hands with 3 others. The fourth person shakes hands with 2 others. The fifth person shakes hands with 1 other. The sixth person has already shaken everyone else's hand. Therefore, the total number of handshakes = 5 + 4 + 3 + 2 + 1 = 15. (For the students with a more extensive background in mathematics, the same result may be found by finding the number of combinations of 6 things 2 at a time ${}_6C_2$, or $\binom{6}{2}$.

14. (A) Let w = the width of the original tin. Let 2w = the length of the original tin.
After forming the box, the length = 2w - 4, the width = w - 4, and the height = 2.
Therefore, the volume of the box is (2w - 4) (w - 4) (2) = 60; w = 7.

15. (E) If R represents the radius of the smaller circle, then 2R is the radius of the larger one. Since the ratio of the radius is $\frac{1}{2}$, the ratio of the areas is $\frac{(1)^2}{(2)^2} = \frac{1}{4}$, and the ratio of the area of the shaded region to the area of the larger circle is $\frac{3}{4}$.

16. (A) Since ΔAPB is isosceles, $\angle PAB = \angle ABP = 30^0$; similarly, since ΔBPC is isosceles, $\angle PCB = \angle PBC = 40^0$. If we let $\angle PAC = x^0$, then $\angle PCA = x^0$ (since ΔAPC is isosceles).
Hence, the sum of the angles of ΔABC is represented by $30^0 + 30^0 + 40^0 + 40^0 + x + x = 180^0$, and $x = 20^0$.

17. (B) The average price paid per pencil = $\frac{\text{total amount of money spent}}{\text{total number of pencils purchased}}$.
At 4¢ per pencil, 75 pencils were purchased, whereas at 6¢ per pencil, 50 pencils were purchased - a total of 125 pencils. Therefore, the average price per pencil = $\frac{\$6.00}{125} = 4.8$.

18. (B) Let the distance to school be 2x feet. Then one half the distance is x feet. At 1 foot per second, Herb could walk x feet in x seconds; at 4 feet per second, in $\frac{x}{4}$ seconds; at 8 feet per second, in $\frac{x}{8}$ seconds. His total walking time is thus $\frac{x}{4}$ plus $\frac{x}{8}$ or $\frac{3x}{8}$ seconds; his total distance, 2x feet. Since average rate equals $\frac{\text{total distance}}{\text{total time}}$, Herb's rate is $\frac{2x}{\frac{3x}{8}}$ feet per second, which reduces to $\frac{16}{3}$, or $5\frac{1}{3}$.

19. (C) If $\angle P = x^0$, then $\angle AOP = x^0$ (since ΔAOP is isosceles). $\angle BAO = \angle AOP + \angle APO = 2x$; therefore, $\angle ABO = 2x$ (since ΔAOB is isosceles). $\angle COB = \angle ABO + \angle APO = 2x + x = 3x$. Since $3x = 60^0$, $x = 20^0$.

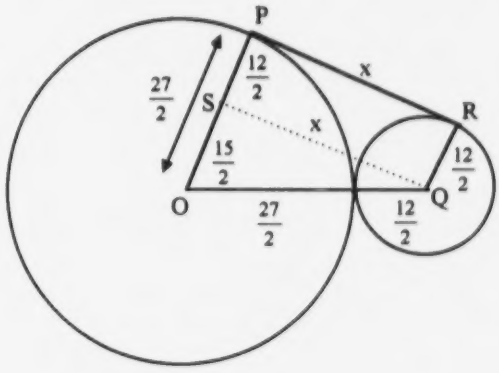
20. (B)

Apply the Pythagorean theorem to ΔQSO:

$x = SQ = \dfrac{36}{2} = 18.$

Or see that the sides of this triangle are multiples of 5, 12, 13;

that is, $\left(\frac{3}{2}\right)(5)$, $\left(\frac{3}{2}\right)(12)$, and $\left(\frac{3}{2}\right)(13)$.



TEST 6 SOLUTIONS

1. (C) One way or the other we can dispose of choice (C): $0 < x$, which we should read as "x is positive". Nothing in the original statement tells us if x is positive or negative. Both are possible since $x < 1$.
Therefore, the answer is (C).

2. (B) If the difference between two numbers is 0, then the two numbers must be equal.
Hence their quotient must be 1.

3. (C) This question can be represented as $5\frac{7}{10} \div \frac{1}{5}$. This equals $\frac{57}{10} \times \frac{5}{1} = 28\frac{1}{2}$.

4. (A) Let x = the number of pens bought for "a" cents. $\dfrac{\text{p cents}}{\text{a cents}} = \dfrac{\text{1 pen}}{\text{x pens}}$. Solving this proportion: $x = \dfrac{a}{p}$.

5. (A) If $\frac{1}{5}$ of the spoilage could be prevented by the new machine, then $\frac{4}{5}$ of the spoilage remains spoiled.

$\frac{4}{5} \cdot \frac{1}{60} = \frac{1}{75}$

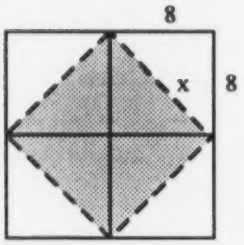
6. (C) Let x = the "other" fraction. The average of the three fractions is $\frac{1}{4} = \frac{1}{3}\left[\frac{1}{3} + \frac{1}{4} + x\right]$. Multiply both sides of the equation by 12: $3 = 4\left(\frac{1}{3}\right) + 1 + 4x$; multiply both sides by 3: $9 = 4 + 3 + 12x$; $x = \frac{1}{6}$.

7. (A) $\dfrac{2a}{b} = \dfrac{7}{1}$. Since $\dfrac{b}{b} = 1$: $\dfrac{2a}{b} - \dfrac{b}{b} = 7 - 1$. Therefore, $\dfrac{2a - b}{b} = 6$.

8. (C) Since there are 60 seconds in 1 minute and 60 minutes in 1 hour, there are (60) (60) seconds in 1 hour.

$\dfrac{\text{5 seconds}}{\text{2 (60) (60) seconds}}(100\%) = \dfrac{5}{72}\%$.

9. (A) If d is the diagonal and s is one side, then the other side is found by the Pythagorean theorem. Let x = the other side: $s^2 + x^2 = d^2$; $x = \sqrt{d^2 - s^2}$. The area then equals: $(s)(x) = s\sqrt{d^2 - s^2}$.



10. (A) Consider the adjoining diagram. By the Pythagorean theorem: $x^2 = 8^2 + 8^2 = 64 + 64 = 128$
$x = \sqrt{128} = \sqrt{64}\sqrt{2} = 8\sqrt{2}$".

11. (C) Q - P + 1; the one is added since Noreen reads the extra page (i.e., page Q).

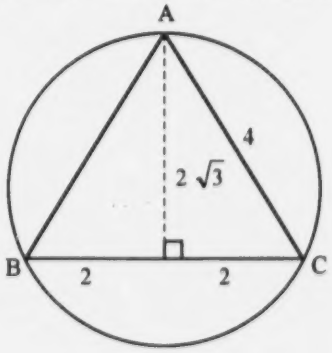
12. (D) $x = \dfrac{6 + 7 + 8}{3} = \dfrac{21}{3} = 7$. The average of *all* the elements is $\dfrac{5 + 6 + 7 + 8 + 7}{5} = \dfrac{33}{5} = \dfrac{66}{10} = 6.6$.

13. (C) Let x = length of shorter dimension.
Set up a proportion: $\dfrac{1\frac{7}{8}}{x} = \dfrac{2\frac{1}{2}}{4}$. Cross multiply: $x\left(2\frac{1}{2}\right) = 4\left(1\frac{7}{8}\right)$; $x = 3$".

14. (C) In order for one number to divide another number exactly, the quotient must be an integer (whole number). The only choices which will be exactly divided by the listed fractions are $\frac{72}{5}$ and $\frac{12}{5}$. The smaller of these is $\frac{12}{5}$.
$\left[\frac{12}{5} \div \frac{6}{5} = 2;\ \frac{12}{5} \div \frac{3}{10} = 8;\ \frac{12}{5} \div \frac{4}{15} = 9\right]$

15. (B) Since ΔAEB is isosceles, $\angle ABE = \angle A$; since ΔACD is isosceles, $\angle ADC = \angle A$. If $\angle A = x^0$, then the sum of the angles of quadrilateral ABFD = $3x^0 + 117^0 = 360^0$, and $x^0 = 81^0$.

16. (B) The points A, B, and C are equally spaced on circle O. Therefore, ΔABC is an equilateral triangle of side 4". By the Pythagorean theorem (ΔADC), the altitude is found to be $2\sqrt{3}$".
Hence the area of $\Delta ABC = \frac{1}{2} \cdot 2\sqrt{3} \cdot 4 = 4\sqrt{3}$.



17. (D) $\dfrac{1}{1 + \dfrac{1}{2 + \frac{1}{3}}} = \dfrac{1}{1 + \dfrac{1}{\frac{7}{3}}} = \dfrac{1}{1 + \frac{3}{7}} = \dfrac{1}{\frac{10}{7}} = \dfrac{7}{10}$

18. (D) Note that the number of coins to be used is not specified; you do not have to use all four. Using one coin only, there are four possible purchases; using two only, there are six; using three, there are again four; and using four, there is only one possibility. The largest number of articles that can be purchased is therefore 15.
Alternate Method: Reason that each of the four coins may be either used or not used (two choices): 2 x 2 x 2 x 2 = 16. However, the possibility in which none of the coins is used does not count, so the answer is 16 - 1 = 15.

19. (C) If x, y, and z represent the times that X, Y, and Z, respectively, can each complete the job working alone, then $\frac{1}{x}$ represents the part of the work completed by X in 1 hour;

$\frac{1}{y}$ represents the part of the work completed by Y in 1 hour;

$\frac{1}{z}$ represents the part of the work completed by Z in 1 hour.

Therefore: $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{1}{8}$ (a)

$\frac{1}{x} + \frac{1}{y} = \frac{1}{12}$ (b)

$\frac{1}{x} = \frac{1}{y} + \frac{1}{z}$ (c)

Substitute (c) into (a): $\frac{1}{x} + \frac{1}{x} = \frac{1}{8}$; $x = 16$ (hours)

Therefore, our answer must be (C).

20. (A) Without loss of generality, we may assume the "star" to be perfectly symmetrical. Consider the circumcircle of the "star": Since each vertex angle is measured by one half of one of the five arcs comprising the entire circle, the sum of these angles equals one half the sum of these arcs; that is, $\frac{1}{2}(360^0) = 180^0$.

Alternate Solution:

Add: $\angle 1 + \angle A + \angle D = 180^0$
$\angle 2 + \angle B + \angle E = 180^0$
$\angle 3 + \angle C + \angle A = 180^0$
$\angle 4 + \angle D + \angle B = 180^0$
$\angle 5 + \angle E + \angle C = 180^0$

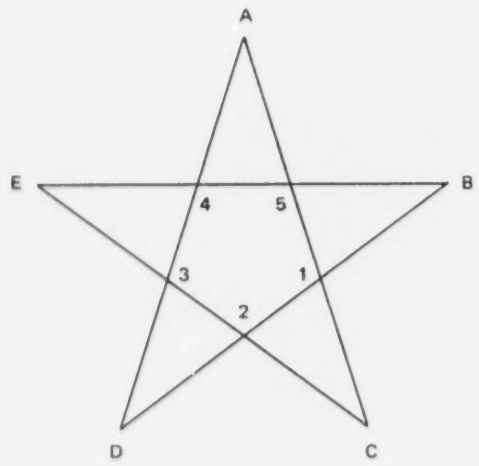
$(\angle 1+\angle 2+\angle 3+\angle 4+\angle 5)+2(\angle A+\angle B+\angle C+\angle D+\angle E) = 5\ (180^0)$

Since a pentagon may be broken down into 3 triangles, the sum of the interior angles of the pentagon is $3\ (180^0)$.

Therefore:

$3\ (180^0) + 2\ (\angle A + \angle B + \angle C + \angle D + \angle E) = 5\ (180^0)$;

$\angle A+\angle B+\angle C+\angle D+\angle E = 180^0$.



TEST 7 SOLUTIONS

1. (A) $14\frac{1}{8} - .25 = \frac{113}{8} - \frac{2}{8} = \frac{111}{8} = 13.875$. Here, you may use a calculator directly.

2. (E) Let x = the missing number. $(7)(3)\left(\frac{1}{5}\right) = \frac{4}{x}$.

Multiply both sides of the equation by 5x: $(7)(3)(x) = (4)(5)$; $x = \frac{20}{21}$.

3. (D) We may begin by clearing the parentheses in choices (B) and (D). This will lead to a quick answer. However, it may be just as quick to factor the original expression ab + cd + bd + ac = a(b + c) + d(b + c), using the two inner and two outer terms. Then factoring (b + c) from both terms gives: (a + d)(b + c).

4. (B) The equation may be written $\frac{t}{800} = \frac{1}{24}$. Cross multiply and solve: $t = 33\frac{1}{3}$.

5. (B) Let D = the depth of the swimming pool. Therefore: C = LWD; $D = \frac{C}{LW}$.

6. (C) $$\frac{352 \text{ steps}}{1 \text{ minute}} = \frac{(352)\left(\frac{3}{2}\right)\text{feet}}{1 \text{ minute}} = \frac{\frac{(352)\left(\frac{3}{2}\right)}{5280}\text{miles}}{\frac{1}{60}\text{hour}} = \frac{352\left(\frac{3}{2}\right)(60)}{5280} = 6\frac{\text{miles}}{\text{hours}}$$

7. (D) $x\sqrt{.04} = .2x = 1$

$2x = 10$

$x = 5$

8. (B) $x^2 + 2xy + y^2 = (x + y)^2 = 2$; therefore, $(x + y) = \pm\sqrt{2}$.

9. (D) Since the area of a right triangle equals one half the product of the legs, $\frac{1}{2}(YZ)(YX) = 60$;

$\frac{1}{2}(8)(YX) = 60$;

$YX = 15$. By the Pythagorean Theorem: $XZ = 17$.

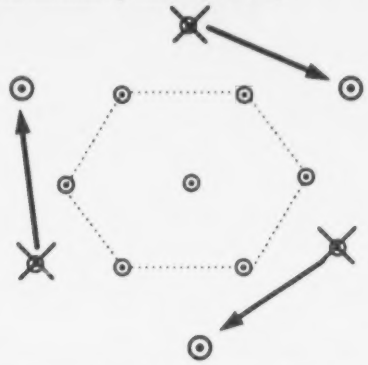
10. (D) The sum of all the horizontal lines is 2b. The sum of all the vertical lines is 2a. Therefore, the perimeter is 2b + 2a = 2(a + b).

11. (C) The length of the rope tangent to one side of the barrels is equal to the distance between the centers of two of the barrels; that is, twice the radius, or 3′. There are three tangents, or 9′. The rope in contact with the curved part of the barrels touches each for $\frac{1}{3}$ the circumference of the barrel, or a total of one circumference, 3π. Thus the whole rope equals 3π + 9 feet.

12. (E) The maximum load is $4\frac{1}{2}$ tons (9000 lb). Therefore, the number of crates is $\frac{9000}{100} = 90$.

13. (E) The formula for finding the area of a triangle = $\frac{1}{2}$(base)(height). (I) Area is unchanged, since base and height are unchanged. (II) & (III) Area is unchanged, since product of base and height is unchanged [i.e., $(\frac{1}{2}b)(2h) = (2b)(\frac{1}{2}h) = bh$].

14. (A) Move only three dots as indicated below (note the symmetry of the center hexagon formed by the dotted line).



15. (B) Let x = the cost of the first book (Donald gained 20%).
Let y = the cost of the second book (Donald lost 20%).

$x + .20\,(x) = \$6.00$
$x = \$5.00$
$y - .20\,(y) = \$6.00$
$y = \$7.50$

The total cost for the two books is x + y = \$12.50. However Donald sold them for \$12.00.
Hence this transaction represent a loss of \$.50.

16. (A) If *l* and w are the dimensions of the base, the area = *l*w. If h and w are the dimensions of the side, the area = hw. If h and *l* are the dimensions of the front, the area = h*l*. $P = (l\text{w})(\text{hw})(\text{h}l) = l^2\,\text{w}^2\,\text{h}^2 = (l\text{wh})^2$.
Since *l*wh = volume, $P = V^2$; $V = \sqrt{P}$.

17. (C) To find the area of the shaded region, subtract the area of the semicircle with diameter 8 from the area of the large semicircle, diameter 12. Then add the area of the semicircle whose diameter is 4.
Therefore, the area of the shaded region is $\frac{1}{2}(6)^2\pi - \frac{1}{2}(4)^2\pi + \frac{1}{2}(2)^2\pi = 18\pi - 8\pi + 2\pi = 12\pi$.

18. (C) From the moment the first car (the engine) of the train enters the tunnel, until the last car leaves the tunnel, 6 minutes (or $\frac{1}{10}$ hour) have elapsed. If the length of the train is x miles, then the length of the tunnel is 2x miles.
Thus the total distance the train travels is $3x = (30)\,(\frac{1}{10})$; x = 1 mile.

19. (B) $\angle B \overset{m}{=} \frac{1}{2}$ (arc CDE); $\angle D \overset{m}{=} \frac{1}{2}$ (arc CBA). Therefore: $\angle B + \angle D \overset{m}{=} \frac{1}{2}$ (arc CDE + arc CBA) =
$\frac{1}{2}(360^0 - \text{arc AE}) = \frac{1}{2}(360^0 - 40^0) = \frac{1}{2}(320^0) = 160^0$.

20. (C) Consider the 8 loaves as part of a "shepherds' fund":

	first shepherd	second shepherd	hunter
amount given to "fund"	3 loaves	5 loaves	no loaves
share taken from "fund"	$\frac{8}{3}$ loaf	$\frac{8}{3}$ loaf	$\frac{8}{3}$ loaf
amount actually left in "fund"	$\frac{1}{3}$ loaf	$\frac{7}{3}$ loaf	

Since hunter pays 8¢ for $\frac{8}{3}$ loaves, each loaf is worth 3¢.

Since each loaf is worth 3¢ (as paid by the hunter), the first shepherd receives $\frac{1}{3}$ of 3¢ or 1¢ for his donation, whereas the second shepherd receives $\frac{7}{3}$ of 3¢ or 7¢ for his donation.

TEST 8 SOLUTIONS

1. (A) $7 + 3(p - q) = 7 + 3(0) = 7 + 0 = 7$

2. (B) Let x = the number to be found. $56 = \frac{7}{8}x; \quad x = 64.$

A more tedious method has you finding $\frac{7}{8}$ of each of the choices.

3. (C) Combining like terms in the following way will save valuable time:

$[(436x - 431x) + 995x] - 1250x = 500$

$[5x + 995x] - 1250x = 500$

$1000x - 1250x = 500$

$-250x = 500$

$x = -2$

4. (B) The original price of Emilie's jeans is \$25.00 + \$2.50 = \$27.50.

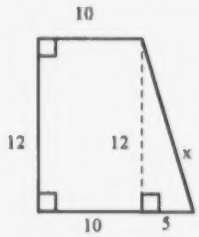
The part of the original price Emilie saved is $\frac{\$2.50}{\$27.50} = \frac{1}{11} \approx 9\%$

5. (D) The even-numbered terms form the following sequence: 8, 6, 4, 2. The odd-numbered terms form the following sequence: 1, 3, 9, 7, a series in which the "9" must be replaced by a "5."

6. (E) Factor the expression in the following way:

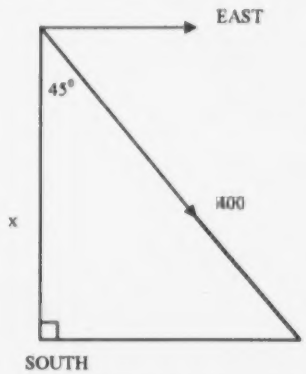
$211\ (555 + 445) + 789\ (555 + 445) = 211\ (\ 1000) + 789\ (\ 1000) = (211 + 789)\ (\ 1000) = (\ 1000)\ (\ 1000) =$ 1,000,000

7. (D) By applying the Pythagorean theorem to the right triangle, we get x = 13.



8. (D) The old price per pound was $\frac{45}{5}¢ = 9¢$. The present price per pound is $\frac{25}{2}¢ = 12\frac{1}{2}¢$. The ratio of the present price per pound to the old price per pound is $12\frac{1}{2}$:9, or 25:18.

9. (B) The adjoining diagram shows the isosceles right triangle to be considered (an exact southeast course forms a 45^0 angle with the north-south line; hence the isosceles right triangle). By the Pythagorean theorem: $x = 200\sqrt{2}$.



10. (D) Since each square has an area of 16, each side equals 4. The perimeter of the figure is composed of $\frac{3}{4}$ of the perimeter of each of the 4 outer squares. Therefore, the perimeter = $4(\frac{3}{4})(16) = 48$. [Or: merely multiply 4 (the length of each side) by 12 (the number of sides of the squares in the perimeter), which equals 48.]

11. (D) Since the ratio of the diameters is $\frac{2}{1}$, the ratio of the circumferences is also $\frac{2}{1}$ (both pairs are corresponding linear parts).

12. (C) Assume that the hour hand will not move. In this case, the passing over will occur at 5:25. However, in actuality, the hour hand will move, so to compensate for this movement we multiply 25 by the "magic fraction," $\frac{12}{11}$, to get $\frac{300}{11} = 27\frac{3}{11}$. Thus the passing over occurs at $5{:}27\frac{3}{11}$.
Alternate Solution: At 5:24 the hour hand will be at the 27 minute mark, since it started from the 25 minute mark and traveled one mark every $\frac{1}{5}$ hour (12 minutes). For the next 12 minutes, the hour hand will be in the interval between 5:27 and 5:28. However, the minute hand will pass through this interval, in this time, thus passing over the hour hand.

13. (C) Let h = the altitude of the triangle; x = the side of the square; then 2x = the base of the triangle. The area of the triangle = $\frac{1}{2}$ (2x)(h) = xh. The area of the square = x^2.
Therefore $xh = x^2$; h = x (making the required ratio $\frac{1}{1}$ or 1).

14. (B) The first sound occurs at 6 o'clock, then an additional sound occurs for each of the next 5 seconds. Hence, there is a 1-second interval between sounds. At 12 o'clock, when the clock sounds 12 times, there are 11 1-second intervals. Thus, the clock strikes for 11 seconds at noon.

15. (B) Since $\frac{\text{area of sector}}{\text{area of circle}} = \frac{\text{number of degrees in central angle}}{360^0} = \frac{\text{length of arc}}{\text{circumference}}$,
$\frac{\text{area of sector}}{\text{area of circle}} = \frac{4}{2\pi\left(\frac{8}{\pi}\right)} = \frac{1}{4}$.

16. (E) Let x = value of stock.
Let $\frac{3}{5}x$ = value of cash sales.
Let $\frac{1}{4}x$ = value of charge account sales.
Let $\frac{3}{20}x$ = value of COD sales.
Then $\frac{3}{20}x = \frac{1}{4}x - 4000$; x = \$40,000; $\frac{3}{5}x$ = \$24,000.

17. (D) Triangles ABC and EBC share the same base (BC); hence their areas are in the same ratio as their altitudes (5:2). Therefore, the ratio of the area of ΔABC to the area of the shaded region is 5:3, or $\frac{5}{3}$.

18. (C) Let $\frac{d}{2\pi}$ = the number of revolutions of the front wheels. Let $\frac{d}{3\pi}$ = the number of revolutions of the back wheels. $\frac{d}{2\pi} = \frac{d}{3\pi} + 10$, thus $d = 60\pi$.

19. (E) $P = a_1 \times a_2 \times a_3 \times \ldots \times a_n$ (a series in which $a_1, a_2, \ldots a_n$ are the n different numbers.) Multiply each member of the product by 5: $5a_1 \times 5a_2 \times 5a_3 \times \ldots 5 \times a_n = \underbrace{5 \times 5 \times 5 \times 5 \times \ldots \times 5}_{n} \times a_1 \times a_2 \times a_3 \times \ldots \times a_n = 5^nP$.

20. (C) Let y = the number of correct answers.
Let 26 - y = the number of wrong answers.

$$xy - 6(26 - y) = 0;\ x = \frac{6(26 - y)}{y}$$

Now substitute each of the choices as values for y in the above equation in order to see which choice satisfies the conditions for x: (A) x = 20 (B) $x = \frac{34}{3}$ (C) x = 7 (D) $x = \frac{22}{5}$ (E) $x = \frac{8}{3}$.
Since x must be an integer, we may only consider (A) and (C). However, x must also be less than 20; therefore, the only answer is (C).

TEST 9 SOLUTIONS

1. (C) By "rounding off" the two numbers, we get: 39,000,000 x 7,000 = 273,000,000,000.
Using a calculator gives: $2.83720598 \bullet 10^{11} = 283,720,598,000$.

2. (A)

$\frac{1}{3} > \frac{2}{9}$, since (9)(1) > (3)(2)
$\frac{1}{3} > \frac{1}{4}$, since (4)(1) > (3)(1)
$\frac{1}{3} > \frac{1}{5}$, since (5)(1) > (3)(1)
$\frac{1}{3} > \frac{4}{15}$, since (15)(1) > (3)(4)

3. (B) Since $(a + b)^2 = a^2 + 2ab + b^2$, then $a^2 + 2ab + b^2 = a^2 + b^2$ only when the middle term of the left side of the equation is eliminated. This occurs only when a = 0 or b = 0.

4. (B) To build the house, (9)(36) "man-days" are required. Let x = the number of days in which 27 carpenters can build the same house. 27x = (9)(36); x = 12

5. (A) The towels can have 11 overlapping edges, each requiring one pin. Each of the two free ends also needs a pin. Hence 13 pins are needed.

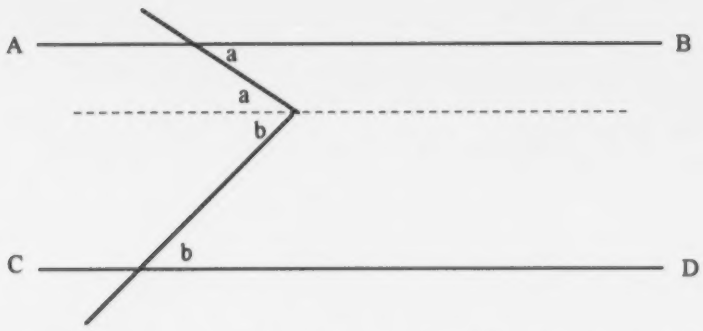
6. (D) Since the ratio of the diameters is $\frac{1}{2}$, the ratio of the areas is $\frac{1}{4}$. Thus 4 small circles will have the same area as the large circle.

7. (D) Since the edge of the cube is 4", the area of one of its faces is $(4)^2 = 16$ square inches.

8.(C) In I., (a + b) + (c + d) = a + (b + c) + d [Law B]
= a + (c + b) + d [Law A]
= (a + c) + (b + d) [Law B]
Therefore, for I. alone, both A and B were needed.

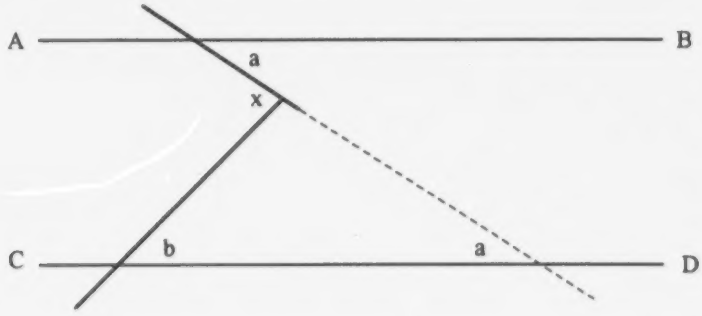
9. (B) If K represents the area of the square, then $\sqrt{K}$ represents the side of the square and the radius of the circle. Therefore, the area of the circle is $\pi(\sqrt{K})^2$ or πK.

10. (E)



Draw a line through the vertex of $\angle x$ and parallel to AB and CD. Note the pairs of equal alternate interior angles marked in the adjoining diagram. Thus $x = a + b$.

Alternate Solution:



Extend one line, as shown in the adjoining diagram. Note the pair of equal alternate interior angles. Since x is an exterior angle of the triangle, $x = a + b$.

11. (B) $(3t - 1)^2 = (3t)^2 - 6t + 1 = 9t^2 - 6t + 1$ (a)

Given: $t^2 = 3$ and $t = \pm\sqrt{3}$ (b)

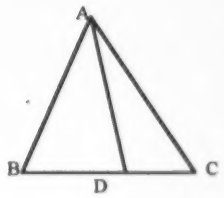
Substitute (b) in (a): $9(3) - 6(\pm\sqrt{3}) + 1 = 27 \pm 6\sqrt{3} + 1 = 28 \pm 6\sqrt{3}$.

12. (E) First, we must find the area of the 6 isosceles right triangles of hypotenuse $\frac{36}{6} = 6$. The sum of the areas of these triangles may be considered equal to the area of 3 squares with diagonal 6. The sum of the areas of these squares $= 3\left[\frac{1}{2}(6)^2\right] = 54$.

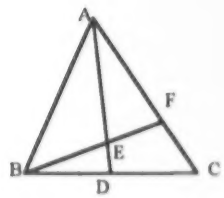
Next, we observe that the area of the hexagon is equal to the area of 6 equilateral triangles with side 6, which is $6\left[\frac{6^2\sqrt{3}}{4}\right] = 54\sqrt{3}$.

Hence the total area $= 54 + 54\sqrt{3}$.

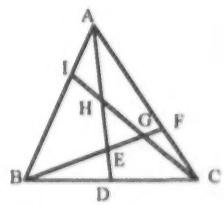
13. (D) The following method of approaching this type of problem has proven quite valuable. First consider the triangle with only one interior line: The different triangles are: ABC, ABD, ADC.



Then add another of the interior lines (BEF) and count all the triangles which depend on this new line for a side. The different triangles are: BED, ABE, ABF, AEF, BFC.



Now add the last of the interior lines (CGHI) and count all the triangles which depend on it for a side. The different triangles are: BGC, BIC, HEG, DHC, BIG, AIH, AHC, GFC, AIC.



In all, there are 17 different triangles.

14. (A) The area of shaded region may be found by subtracting the area of four quarter circles (each with radius s) from the area of the square. Therefore, the area of the shaded region = $(2s)^2 - \pi s^2 = 4s^2 - \pi s^2 = s^2(4 - \pi)$.

15. (C) In order for a number to be divisible by 9, the sum of its digits must be divisible by 9. (See rules for divisibility on page 8). Hence a + 4 + 7 + b (which equals a + b + 11) must be divisible by 9. If a + b = 7, then the sum a + b + 11 = 7 + 11 = 18, which is divisible by 9.

16. (C) Originally, Harvey owes Al 10¢. If Al gives Harvey 60¢, then Harvey owes Al 60¢ more than before, or 70¢. To cancel the debt, Harvey must give Al 70¢.

17. (B) A good approach to the problem is to assume that there are 100 students in the school. In this case, 60 students are boys. $\frac{1}{20}$ of these, or 3, plan to be doctors. The number 3 is $\frac{3}{4}$ of all who plan to be doctors; therefore, there are a total of 4 planning to be doctors: 3 boys and 1 girl. (There are 40 girls in the school, so that $\frac{1}{40}$ of them plan to be doctors.)

The result does not depend on the number of students assumed to be in the school, but you have to employ foresight to simplify the arithmetic. Of course, you can always change the assumed number (100) and start over if you encounter awkward fractions while solving the problem.

18. (B) Let 5x = the number of soldiers the first army had before the battle. Let 6x = the number of soldiers the second army had before the battle.

$$\frac{5x - 14000}{6x - 6000} = \frac{2}{3};\ x = 10{,}000.$$

The first army had 5x - 14000 = 36,000 soldiers after the battle; the second army had 6x - 6000 = 54,000 soldiers after the battle.

19. (C) Since, in one revolution, the slab moves π feet relative to the roller while the roller moves π feet relative to the ground, the slab moves 2π feet relative to the ground.

20. (E) Let t_N = Noreen's time. Let t_A = Al's time.
Let R = Al's rate of walking.

$$t_A = \frac{12}{R}$$

$$t_N = \frac{12}{R} - 2$$

$$(\text{new } t_A) = \frac{12}{2R}$$

Therefore: $\frac{12}{2R} = \left(\frac{12}{R} - 2\right) - 1.$

To solve this equation, multiply both sides by 2R (the lowest common denominator): $12 = 24 - 6R$; $R = 2$ miles per hour.

TEST 10 SOLUTIONS

1. (B) If a number is divisible by 2, 3, and 5, then it is divisible by 2 X 3 X 5 or 30. Therefore, the only numbers less than 100 divisible by 2, 3, and 5 are 30, 60, and 90.

2. (D)

$$\frac{3}{8} > \frac{4}{11}, \text{ since } (3)(11) > (4)(8)$$

$$\frac{4}{11} > \frac{1}{3}, \text{ since } (4)(3) > (1)(11)$$

In a similar way, we may see that the other choices are not arranged in descending order.

3. (D) Consider the differences between consecutive numbers in our original series: 3, 4, 5, 6. Therefore, the next difference must be 7, so that the next number in the series is 20 + 7 = 27.

4. (D) We must eliminate "3" as a possible value for x, since that number would give us 0 in the denominator (division by 0 is undefined). However, x = -3 yields $\frac{0}{-6} = 0$

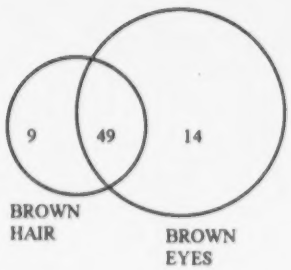
5. (C) $\frac{1 - a^2}{a - 1} = \frac{(1 - a)(1 + a)}{-(1 - a)} = -(1 + a) = -1 - a.$

6. (E) Let -x be any negative real number, and $-\frac{1}{x}$ its reciprocal. The sum of these $\left[-x + \left(-\frac{1}{x}\right) = \frac{-(x^2 + 1)}{x}\right]$ is greatest when $\frac{x^2 + 1}{x}$ is smallest (remember: x > O). This greatest sum is achieved when x = 1.

Therefore, $\left(\frac{x^2 + 1}{x} = \frac{1 + 1}{1} = 2\right)$.

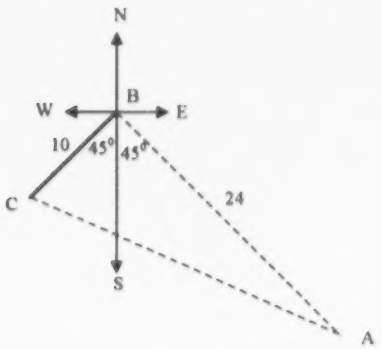
7. (E)

In the adjoining Venn diagram, we see that 9 people have brown hair and nonbrown eyes. 14 people have brown eyes and nonbrown hair. 49 people have both brown hair and brown eyes. Since there are 87 people in the group, 15 people have neither brown hair nor brown eyes [87 - (9 + 14 + 49) = 87 - 72 = 15].



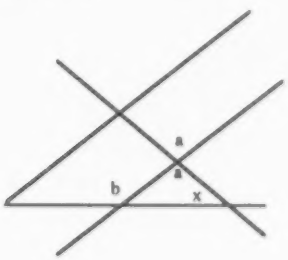
8. (D) If $x! = x(x - 1)(x - 2)(x - 3) \ldots (3)(2)(1)$,
then $4! = (4)(3)(2)(1) = 24$.

9. (A)
Since $AB \perp BC$, we may apply the Pythagorean theorem to right ΔABC to get $10^2 + 24^2 = (AC)^2$, and $AC = 26$.



(Note: This is a 5, 12, 13 right triangle.)

10. (B)
Since $\angle b$ is the exterior angle of the triangle, $b = a + x$.
Therefore, $x = b - a$.



11. (C) If $90 = \frac{n^2 - 3n}{2}$, then $n^2 - 3n - 180 = 0$.

$(n - 15)(n + 12) = 0$

$n = 15$ | $n = -12$

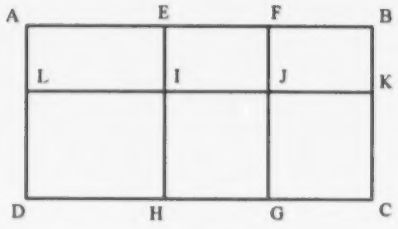
| We reject this value, since "n" cannot be negative.

12. (D) If the ratio of the diameters is $\frac{1}{3}$, then the ratio of the areas is $\frac{1^2}{3^2} = \frac{1}{9}$.

13. (E)

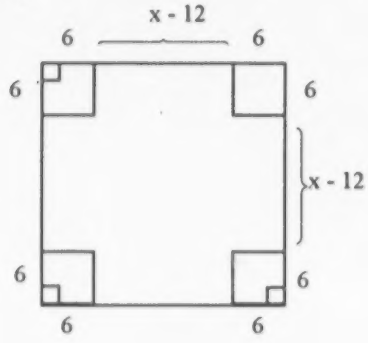
The following is a list of all the rectangles:

AEIL	LIHD	AEHD
AFJL	LJGD	EFGH
ABKL	LKCD	FBCG
EIJF	IHGJ	ABCD
EIKB	IHCK	AFGD
FJKB	JGCK	EBCH



14. (A) If D = the diameter of the original circle, then πD is the circumference of the original circle. The new circle's circumference = $\pi(D + \pi) = \pi D + \pi^2$, or an increase of π^2 over the original circle's circumference.

15. (B)



If x = the side of the square, then one side of the newly formed box = x - 12. Hence the volume of the box is

$(x - 12)^2(6) = 150$

$(x - 12)^2 = 25$

$x - 12 = 5$

$x = 17$

16. (C) Out of the first 28 days in any month, 20 are school days. A 31-day month can have a maximum of 23 school days, and a 30-day month can have a minimum of 20 school days if the two "extra" days comprise a weekend.

17. (E) Pages 1-9 use 9 digits.
Pages 10-99 use (90)(2) digits, or 180 digits.
Pages 100-999 use (900)(3) digits, or 2700 digits.
At this point, 2700 + 180 + 9 = 2889 digits of the 3357 digits have been used to number pages 1-999. Remaining are 468 digits, which can be used to form 117 four-digit numbers. Therefore, the total number of pages is 999 + 117 = 1116.

18. (A) Let *l* and w be the length and width, respectively, of one of the 5 congruent rectangles. Since two lengths equal three widths, the ratio $\frac{l}{w} = \frac{3}{2}$. The area of each of the 5 congruent rectangles is $\frac{30}{5} = 6$. Hence $lw = 6$. Therefore, $l = 3$ and w = 2, and the perimeter of rectangle ABCD = $2[2l + (l + w)] = 2[2(3) + (3 + 2)] = 2[6 + 5] = 22$.

19. (D) Assume that the hour hand does not move. In that case, the minute hand would have to travel 40 minutes to the "8" to be perpendicular to the hour hand (after 5:30). However, the hour hand does not remain stationary. Therefore, in order to allow for the movement of the hour hand, we must multiply 40 minutes by the "magic fraction," $\frac{12}{11}$.

$$40 \bullet \frac{12}{11} = \frac{480}{11} = 43\frac{7}{11}$$

Thus, the time when the hands are perpendicular is $5{:}43\frac{7}{11}$.

20. (D) Let u = the number of unskilled workers and s = the number of skilled workers. In the following, we equate the number of men in each group:

$.80u + .90s = .82(u + s)$

$u = 4s$

Thus $\frac{s}{u} = \frac{1}{4}$.

TEST 11 SOLUTIONS

1. (C) Since there are an even number of negative terms, the answer will be positive. Consider the following cancellations:

$$\frac{(12)(21)^3}{(6)(1)(7)(4)(3)} = \frac{1}{2}$$

2. (B) (A) .1 (B) .008 (C) .5 (D) .05 (E) .01
Thus (B) has the smallest value.

3. (B) Change each choice to decimal form, then compare each to the decimal equivalent of $\frac{4}{7}$. A calculator is useful here.

(A) $\frac{5}{9} = .5555\overline{5}$ (B) $\frac{7}{12} = .583333\overline{3}$ (C) $\frac{6}{11} = .545454\overline{54}$ (D) $\frac{3}{5} = .60$ (E) $\frac{1}{2} = 50$

$\frac{4}{7} = .571428\overline{571428}$: a number which is closest to (B).

4. (A) Freddy's original rate is $\frac{15}{5}$ or 3 miles per hour. His new rate is 3 or 5 miles per hour, an increase of 2 miles per hour.

5. (C) A cubic foot = (12) (12) (12) cubic inches, and 2 cubic inches weigh 1 ounce. Therefore:

$$\frac{1 \text{ ounce}}{1 \text{ cubic foot}} = \frac{2 \text{ cubic inches}}{(12)(12)(12) \text{ cubic inches}} = \frac{1}{864}.$$

6. (E) In order for a triangle to exist, the sum of any two sides must be greater than the third.
Therefore, $7+5 > x$, $5 + x > 7$, and $7 + x > 5$. Thus $2 < x < 12$ (using the first two inequalities).

7. (E) $4s = 3t$; $s = \frac{3t}{4}$.

8. (E) Set up the following proportion:

$$\frac{\frac{5}{8}\text{ gallon}}{\frac{3}{5}\text{ gallon}} = \frac{\$2.75}{x} \text{ (where x = cost of } \frac{3}{5} \text{ gallon of soap)}$$

$$\frac{25}{24} = \frac{\$2.75}{x}$$

Cross multiply and solve: x = $2.64.

9. (E) The tax on the first 24 horsepower = ($.30)(24) = $7.20. The remaining money ($2.80) is absorbed by 7 additional horsepower (at 40¢ each). Therefore, the total horsepower = 24 + 7 = 31.

10. (E) $\frac{4}{5}$ of the remaining votes ($\frac{1}{4}$ of the votes) were for Candidate B. Therefore, $\frac{4}{5} \bullet \frac{1}{4} = \frac{1}{5}$ of the votes were for Candidate B. $\frac{3}{4} + \frac{1}{5} = \frac{19}{20}$ of the votes were for Candidates A and B. Hence $\frac{1}{20}$ of the votes were for the other candidates.

11. (B) The sequence follows the following pattern:

Computation of sequence	2(0)	2(0)+1	2(1)	2(1)+1	2(3)	2(3)+1	2(7)	2(7)+1	2(15)	2(15)+1	2(31)	2(31)+1
Sequence	0	1	2	3	6	7	14	15	30	31	62	63

12. (C) The sum of the angles in the large triangle is 2x + 2y + z. The sum of the angles in the small triangle is 2x + ?. Since the sum of the angles of all triangles is a constant, ? = 2y + z.

13. (E) Since choice (C) contradicts choice (E), only one of the two may be correct. Let us then check commutativity.

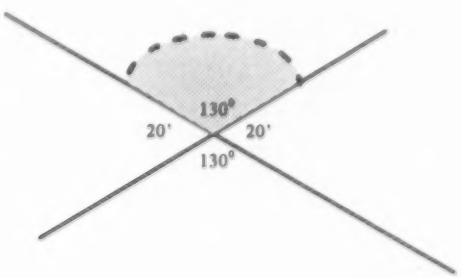
$2 * 3 = 2 + 3\,(3) = 11$

$3 * 2 = 3 + 3\,(2) = 9$

Therefore, it is false that "the operation (*) is commutative."

14. (E) The shaded region (a sector of a circle of radius 20) in the adjoining diagram represents the area in which the cow can roam.

The area of this sector = $\frac{130^\circ}{360^\circ} \bullet \pi R^2 = \frac{1300\pi}{9}$ square feet.



15. (D) Let $y = \frac{1}{2}$ the third side of the triangle. By the Pythagorean theorem:

$$x^2 + y^2 = (2x)^2$$
$$y = x\sqrt{3}$$

Therefore, the third side (2y) equals $2x\sqrt{3}$.

16. (B) Let x = the number of square inches representing 20 acres on the scale drawing. 20 acres = 3200 square rods. Set up the following proportion:

$$\frac{\left(\frac{1}{8}\right)^2 \text{ square inch}}{(1)^2 \text{ square rod}} = \frac{x \text{ square inches}}{3200 \text{ square rods}}$$

Cross multiply: x = 50.

17. (C) Let x = the number of hours it takes 16 typists to type 384 pages. Set up the following proportion:

$$\frac{240 \text{ pages}}{384 \text{ pages}} = \frac{(15)(8) \text{ typist - hours}}{(16)(x) \text{ typist - hours}}$$
$$x = 12$$

18. (B) Since arc AB = 60^0 or $\frac{1}{6}$ of the complete circle, the area of the shaded region is $\frac{1}{6}$ the area of the "ring" (the area between the two circles). The area of the ring is found by subtracting the areas of the two circles: $64\pi - 36\pi = 28\pi$. Hence the area of the shaded region is $\frac{1}{6}(28\pi) = 4\frac{2}{3}\pi$ square inches.

19. (A) Since 4 D. Marks = 25 Schillings

$$1 \text{ D. Mark} = \frac{25}{4} \text{ Schillings}$$
$$3 \text{ D. Marks} = \frac{75}{4} \text{ Schillings}$$

60 Pfennigs = $\frac{3}{5}$ D. Mark = $\frac{3}{5} \cdot \frac{25}{4}$ Schillings = $\frac{15}{4}$ Schillings

Therefore, 3D.-Marks + 60 Pfennigs = $\frac{75}{4}$ Schillings+ $\frac{15}{4}$ Schillings = $22\frac{1}{2}$ Schillings = 22 Schillings + 50 Groschens.

20. (D) 25 Schillings = 4 D. Marks

$$1 \text{ Schilling} = \frac{4}{25} \text{ D. Marks}$$
$$192 \text{ Schillings} = 192\left(\frac{4}{25}\right) \text{ D. Marks} = \frac{768}{25} \text{ D. Marks}$$

50 Groschens = $\frac{1}{2}$ Schilling = $\frac{1}{2} \cdot \frac{4}{25}$ D. Marks = $\frac{2}{25}$ D. Marks

Therefore, 192 Schillings + 50 Groschens = $\frac{768}{25}$ D. Marks + $\frac{2}{25}$ D. Marks = $\frac{770}{25}$ D. Marks = $30\frac{4}{5}$ D Marks = 30 D. Marks + 80 Pfennigs.

TEST 12 SOLUTIONS

1. (D) Since all four numbers are the same at the beginning, 971.0049, we need only to inspect the remaining digits of each number. The order then is 2 - 4 - 1 - 3.

2. (A) Since the two largest two-digit numbers (99 and 99) have a sum of 198, the first digit of the sum can never exceed 1.

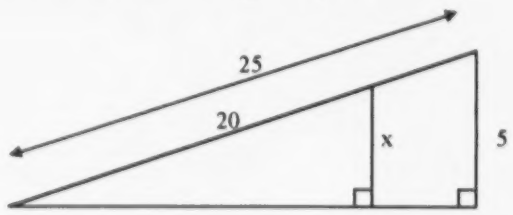
3. (B) Since the inch scale has each unit represent 2 inches and the centimeter scale has each unit represent $2\frac{1}{2}$ centimeters, the line crosses 5 inches at $12\frac{1}{2}$ centimeters.

4. (E) 1 bushel = 4 pecks.

$\frac{1}{4}$ bushel = 1 peck = 8 quarts.

Therefore, 10 quarts is $\frac{10}{8}$ (100%), or 125% of $\frac{1}{4}$ bushel.

5. (D) Let x = the height achieved after 20 feet. Since the triangles are similar in the adjoining diagram, their corresponding sides are in the following proportion: $\frac{20}{25} = \frac{x}{5}$; x = 4.



6. (A) If A, B, and C are the angles of the triangle, then $A + B + C = 180^0$ and $A + B = C$.
By substituting for C in the first equation, we get $2C = 180^0$; $C = 90^0$. Hence the triangle must be a right triangle.

7. (A) With new rings, she can go 120 miles on .8 quarts. Let x = the number of miles she can go on one quart of oil. Set up the following proportion: $\frac{120 \text{ miles}}{x \text{ miles}} = \frac{.8 \text{ quarts}}{1 \text{ quart}}$; x = 150.

8. (C) Let x = the number of degrees it travels in 1 second. Set up the following proportion:

$$\frac{75 \text{ seconds}}{1 \text{ second}} = \frac{360^\circ}{x} \quad (1\tfrac{1}{4} \text{ minutes} = 75 \text{ seconds})$$

Cross multiply and solve: $x = 4\frac{4}{5}$ degrees.

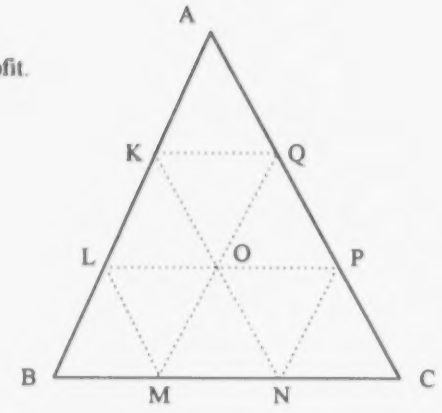
9. (C) If it takes a plane 5 hours to travel 1000 miles, its rate is $\frac{1000}{5}$ or 200 miles per hour. Similarly, if it takes $4\frac{1}{2}$ hours to travel 1000 miles, its rate is $\frac{1000}{4\frac{1}{2}}$ or $222\frac{2}{9}$ miles per hour, an increase of $22\frac{2}{9}$ miles per hour.

10. (B) (S - 18) is the profit

$\frac{S - 18}{S}$ is the *part* of the selling price which is profit.

$\frac{100\,(S - 18)}{S}$ is the percent of the selling price which is profit.

11. (C)
Draw dotted lines, as in the adjoining diagram. Since ΔABC is composed of 9 congruent equilateral triangles and hexagon KLMNPQ is composed of 6 congruent equilateral triangles, the ratio of the areas of these figures is 3:2, or $\frac{3}{2}$, respectively.



12. (A) Since $DF = \frac{5}{4}(CF)$, $DC = DF + CF = \frac{5}{4}(CF) + CF = \frac{9}{4}(CF)$. However, DC = 27.

Therefore: $\frac{9}{4}(CF) = 27$; CF = 12. Hence ΔBCF is an isosceles right triangle, and $\angle FBC = 45^0$.

13. (D) Apply the Pythagorean theorem to the following triangles, in the order in which they are listed: ΔOAB, ΔOBC, ΔOCD, ΔODE. In this way, we find the lengths of OB, OC, OD, and OE to be $\sqrt{2}$, $\sqrt{3}$, $\sqrt{4}$, and $\sqrt{5}$, respectively.

14. (D) Let x = the mark Lynn must earn in her mathematics course in order to achieve an overall average of 90%.

$$\frac{87 + 93 + 96 + 85 + 88 + x}{6} = 90$$

$$x = 91$$

15. (C) In order to find one discount equivalent to two successive discounts, subtract each of the discounts (in decimal form) from 1.00, multiply the resulting differences, and subtract this product from 1.00. For example, one discount equivalent to successive discounts of 10% and 20% is found in the following way:

$$\left.\begin{array}{l} 1.00 - .10 = .90 \\ 1.00 - .20 = .80 \end{array}\right\} \text{multiply to get: } .72$$

$$\text{then: } 1.00 - .72 = .28 \text{ or } \underline{\underline{28\%}}$$

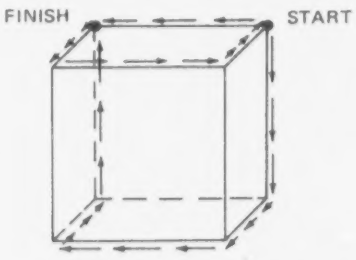
In a similar way, we find successive discounts of 15% and 15% to equal one discount of 27.75%. Therefore, the discount at "A" exceeds the discount at "B" by .25% or $\frac{1}{4}$ %.

16. (C) Edith sells (13)(22) or 286 quarts of milk daily. Since each quart weighs $1\frac{1}{2}$ pounds, Edith sells (286) $(1\frac{1}{2})$ or 429 pounds of milk daily. However, 17 cows will only produce 425 pounds of milk (25 pounds of milk each). Therefore, she will need at least 18 cows.

17. (B) By means of the definition given, we get the following equation: (3)(-5) - (7)(x) = 6; x = -3.

18. (E) What we seek in this problem is the least common multiple of 7 and 3 (i.e., 21). Since the radii are in the ratio 7: 3, the circumferences are also in the ratio 7:3. Hence when the large circle makes 3 revolutions, it travels as much as when the small circle makes 7 revolutions.

19. (D) There are various ways the bug may travel and still cover the greatest number of sides. One of these ways may be seen in the diagram below:



20. (B) Let x = the total number of dollars of the check. $\frac{x}{8}$ = the number of dollars each person paid in the first case. $\frac{x}{7}$ = the number of dollars each person paid in the second case. Therefore, $\frac{x}{8} + 20 = \frac{x}{7}$; x = \$11.20.

TEST 13 SOLUTIONS

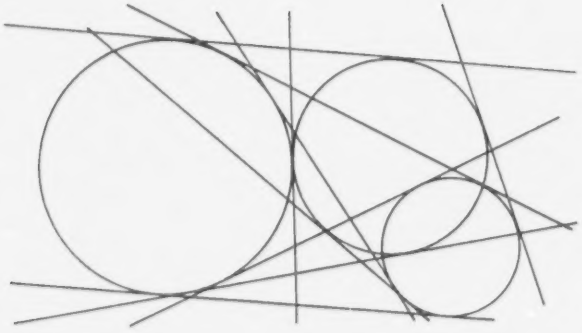
1. (E) Let x = the number to be found. $\frac{1}{8}\%(x) = 5$, or $\left(\frac{1}{8}\right)\bullet\left(\frac{1}{100}\right)\bullet x = 5$; x = 4,000. You can also use a calculator and multiply each choice by .00125.

2. (A) Factor the numerator: $\frac{2 \bullet 3 \bullet 4(1 + 5 \bullet 7 \bullet 2)}{2 \bullet 3 \bullet 4(5 \bullet 7 \bullet 2)} = \frac{71}{70}$. If you use a calculator, be careful to observe the order of operations and do the multiplying first.

3. (A) If a > b and b > 0, then a > 0. However, ac < 0; therefore, c < 0 (since the product of a positive number and a negative equals a negative number).

4. (E) Since the ratio of two corresponding lengths is $\frac{2}{3}$, the ratio of corresponding areas is $\frac{2^2}{3^2} = \frac{4}{9}$; that is, if an object had an area 4 and now has an area of 9, it has increased its area by 125%.

5. (C) (See adjoining diagram)



There are 2 common tangents for the intersecting circles.
There are 3 common tangents for the tangent circles.
There are 4 common tangents for the nonintersecting circles.
In all, there are 9 common tangents.

6. (C) Substitute and subtract:

Substitute: $41 = 270r + k$

$49 = 280r + k$

Subtract: $8 = 10r$; $r = \frac{4}{5}$

7. (B) The volume of a cube 24 inches on each side is $(24)(24)(24) = (1728)(8)$.
Therefore, 1728 cubes of volume 8 (2 inches on each edge) will fit into the first cube.

8. (D) The shirt was reduced by \$.52. This represents a $\frac{.52}{2.50}(.100)$ percent, or 20.8% reduction.

9. (C) Since the ratio of women to men is 2:3 and the ratio of men to children is 1:2 (or 3:6), then the ratio of women to men to children is 2:3:6.

Hence: $2x + 3x + 6x = 396$; $x = 36$.

The number of men $(3x) = 108$.

10. (A) If x is equal to zero, then the average of the 5 numbers would be 6. However, x must be greater than zero. Therefore, the average of the 5 numbers cannot be 6.

11. (E) The radius of the semicircle (center P) equals the diameter of circle O (i.e., 4). The area of the shaded region is found by subtracting the area of circle O from that of semicircle P. Therefore, the area of the shaded region = $8\pi - 4\pi = 4\pi$.

12. (C) When the two cubes are glued together, we only have to paint 5 sides of each cube, or 10 sides in all. We may then set up the following proportion. Let x = the number of quarts of paint needed for 10 sides.

$$\frac{6 \text{ sides}}{10 \text{ sides}} = \frac{\frac{1}{2} \text{ quart of paint}}{x \text{ quarts of paint}}; \quad x = \frac{5}{6}$$

13. (A) Since the square root of a sum of two numbers is not equal to the sum of the square roots, we must first add the fractions (leaving them in factored form) and then extract the square root:

$$\sqrt{\frac{n^2}{36} + \frac{n^2}{64}} = \sqrt{\frac{64n^2 + 36n^2}{(36)(64)}} = \sqrt{\frac{100n^2}{(36)(64)}} = \frac{10n}{(6)(8)} = \frac{5n}{24}$$

14. (D) When two circles intersect, there are at most two points of intersection. When a third circle intersects each of the first two, it meets them in at most two points each, or 4 additional points (i.e., three circles intersect in at most 6 points). A fourth circle meets each of the previous three circles at most twice, for 6 additional points (i.e., four circles meet in at most 12 points). A fifth circle meets each of the previous four circles at most twice, for an additional 8 points. Therefore, five circles intersect in at most 20 points. The table below is a summary of the above explanation:

number of circles	*maximum number of points of intersection*
2	2
3	2 + 2(2) = 6
4	6 + 2(3) = 12
5	12 + 2(4) = 20

15. (D) Let x = the amount of water being evaporated. $\frac{P}{100}(G)$ = the amount of sugar in the original solution of G gallons. $\frac{S}{100}(G - x)$ = the amount of sugar in the solution where the water has been evaporated. Since the amount of sugar remains unchanged,

$$\frac{P}{100}(G) = \frac{S}{100}(G - x)$$

$$PG = SG - Sx$$

$$x = \frac{G(S - P)}{S}$$

16. (B) Let C = the cost of the fur coat; the marked price = $C + \frac{2}{3}C = \frac{5}{3}C$; the selling price = $C + \frac{1}{3}C = \frac{4}{3}C$. We seek the fraction $\frac{\frac{4}{3}C}{\frac{5}{3}C} = \frac{4}{5} = 80\%$. Since the selling price is 80% of the marked price, the purchaser received a discount of 20% on the marked price. (Note: Selling price was not needed.)

17. (B) Let the initial letter of each boy's name represent the number of marbles he has.

Therefore: A + E = 14 (a)
E + M = 10 (b)
A + M = 12 (c)

Subtract (b) from (c) to get:

A - E = 2 (d)
A + E = 14 (a)

Add (d) and (a) to get: A = 8. It then follows (by substitution) that E = 6, and M = 4. The greatest is 8.

18. (B) Successive discounts of $12\frac{1}{2}\%$ and 20% are equivalent to one discount of 30%. Hence the second discount (32%) exceeds the first discount (30%) by 2%.

19. (A) Since AE bisects $\angle BAC(123^0)$, $\angle BAE = \frac{1}{2}(123^0) = 61\frac{1}{2}^0$. However, since ΔADB is a right triangle, $\angle BAD$ is complementary to $\angle B$ and is therefore equal to 58^0. $x = \angle DAE = \angle BAE - \angle BAD = 61\frac{1}{2}^0 - 58^0 = 3\frac{1}{2}^0$.

20. (B) Assume that the hour hand remains stationary, The minute hand would then have to travel 10 minutes to the "2" to overtake the hour hand. To account for the actual movement of the hour hand, multiply 10 minutes by the "magic fraction," $\frac{12}{11}$, to get $10 \bullet \frac{12}{11} = \frac{120}{11} = 10\frac{10}{11}$. Hence the minute hand will overtake the hour hand at $2{:}10\frac{10}{11}$ (i.e., in $10\frac{10}{11}$ minutes).

TEST 14 SOLUTIONS

1. (C) It is more meaningful to look at the interval given in terms of fractions with a common denominator. Thus the given interval is $-2\frac{1}{2} = -\frac{5}{2} = -\frac{15}{6}$ to $1\frac{1}{3} = \frac{4}{3} = \frac{8}{6}$.
We seek $\frac{3}{8}$ the distance from $-\frac{15}{6}$ to $\frac{8}{6}$, or $\frac{3}{8}$ of $\frac{23}{6} = \frac{23}{16}$. When added to $-\frac{15}{6}$, it is
$\frac{23}{16} + \frac{-15}{6} = \frac{69}{48} - \frac{120}{48} = \frac{-51}{48} = -1\frac{1}{16}$.

2. (D) $\frac{1}{4}\% = \frac{1}{4} \bullet \frac{1}{100} = \frac{1}{400}$. If $\frac{1}{400}$ is acid, then $1 - \frac{1}{400} = \frac{399}{400}$ is *not* acid.

3. (C) 60% of 1 foot = $\frac{x}{12}$ of 3 feet, or $\frac{3}{5} \bullet 1 = \frac{3x}{12}$. Thus $x = \frac{12}{5}$, or $2\frac{2}{5}$.

4. (E) By letting a "B" truckload equal x, we find that the load for 20 trucks is 20x.
Since each "A" truck carries 1.4x, $\frac{20}{1.4}$ or 15 trucks are needed.

5. (D) Let $\angle a = x$ degrees.
Then $\angle b = 2x$ degrees.
$\angle c = 3x$ degrees.
$\angle d = 4x$ degrees.
Since AB is a straight line, $x + 2x + 3x = 180^0$; $x = 30^0$. Therefore, $\angle d = 4x = 120^0$.
However, $\angle e$ is supplementary to $\angle d$; hence $\angle e = 180^0 - \angle d = 180^0 - 120^0 = 60^0$.

6. (E) Since the cans have equal volumes and the heights of the cans are in the ratio 5:4, the ratio of the areas of their bases is 4:5. (This is so because the volume equals the product of the height and the area of the base). If the ratio of their areas is 4:5, then the ratio of their radii (corresponding linear parts) is $\sqrt{4} : \sqrt{5}$, or $2 : \sqrt{5}$.

7. (A) The average of the first 3 elements is 7 (the middle number). Hence we seek the average of the numbers 6, 7, 8, and 7. This average is 7.

8. (C) $\frac{1}{x} + \frac{1}{x} + \frac{1}{x} + \frac{1}{x} = 4\left(\frac{1}{x}\right) = \frac{4}{x}$. Substituting $2 + 2k$ for x yields: $\frac{4}{2 + 2k} = \frac{4}{2(1 + k)} = \frac{2}{1 + k}$.

9. (D) In order for a square to have an area of 25 square units, each side would have to be of length 5 units. If the sides of the square are parallel to the x and y axes, then the coordinates of the given point would each have to differ from each of the corresponding coordinates of the point diagonally opposite it by 5 units. Since $|7 - 2| = 5$ and $|-1 - 4| = 5$, choice (D) must be the desired point.

10. (C) If there are 12.8 TVs per 100 persons, then there are 128 TVs for each 1,000 persons. Hence the population of Anyville is $\left(\frac{48{,}000}{128}\right)(1000) = 375{,}000$.

11. (B) $a = \frac{np}{n + 1}$

Multiply both sides of the equation by $(n + 1)$: $a(n + 1) = np$

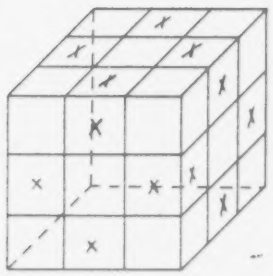
$an + a = np$

$a = np - an$

Factor: $n(p - a) = a$

Divide both sides of the equation by $(p - a)$: $n = \frac{a}{p - a}$

12. (D) Consider the adjoining diagram. The cubes with only two painted faces are those indicated by "x." There are 12 such cubes.



13. (E) The amount of acid in the original solution is .10(12) or 1.2 quarts. The percentage of acid in the second solution is $\left(\frac{1.2}{16}\right)(100\%) = 7.5\%$.

14. (C) What we must compare here are the areas of the cross sections. The ratio of the areas is equal to the ratio of the diameters squared. We may then set up the following proportion. Let d = the required diameter.

$$\frac{d^2}{a^2} \text{ (the ratio of the areas)} = \frac{1 \text{ (fraction of necessary water)}}{\frac{3}{4} \text{ (fraction of necessary water)}}$$

Cross multiply: $\frac{3}{4} d^2 = a^2$; $d = a\sqrt{\frac{4}{3}} = \frac{2a}{\sqrt{3}}$

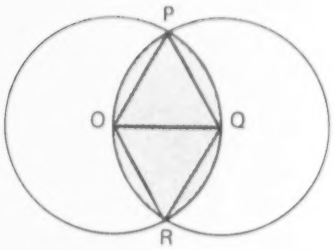
Rationalize the denominator: $d = \frac{2a}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2a\sqrt{3}}{3}$

15. (B) Let $\angle PST = y$. Since ΔSPT is isosceles, $\angle PTS = \angle PST = y$; $\angle SPT = 180^0 - 2y$. $\angle QPR = \angle QPS + \angle SPT = 30^0 + (180^0 - 2y) = 210^0 - 2y$. Since ΔQPR is isosceles, $\angle PQR = \angle PRQ = \frac{1}{2}[180^0 - (210^0 - 2y)] = -15^0 + y$.

$\angle PTS$ is the exterior angle of ΔSTR; therefore, $\angle PTS = \angle TSR + \angle TRS$. By substituting the appropriate values, we get: $y = x + (-15^0 + y)$; $x = y - (-15^0 + y) = y + 15^0 - y = 15^0$.

16. (D) $\frac{1}{3}$ man-hour per employee is the same as 1 man-hour per $\frac{1}{3}$ of the employees. \$1.25 saved from each of $\frac{1}{3}$ of 450 employees (150 employees) equals \$187.50 each day. Since the expert charged \$24 a day, she saved the company \$163.50.

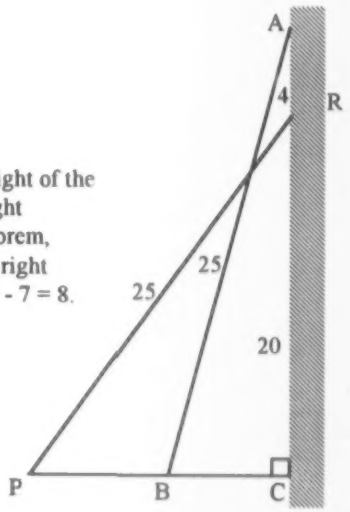
17. (D)



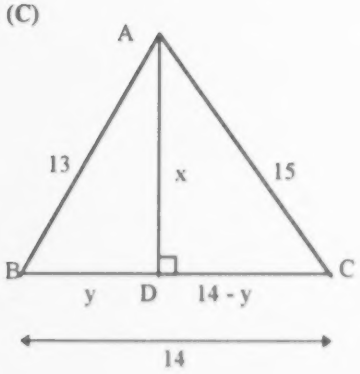
In the above diagram, all the lines are equal. Therefore, ΔOPQ and ΔORQ are equilateral, and $m\angle POQ = m\angle ROQ = 60^0$. The shaded region is composed of twice the area of sector POQ plus twice the area of segment PQ (i.e., the area of segments PO and RO). The area of sector $POQ = \frac{60^0}{360^0}\,\pi(36) = 6\pi$.

The area of segment PQ = the area of sector POQ - the area of $\Delta POQ = 6\pi - 9\sqrt{3}$.
Hence the area of the shaded region equals $12\pi + 12\pi - 18\sqrt{3} = 24\pi - 18\sqrt{3}$.

18. (D) By employing the Pythagorean theorem in ΔABC, we find the height of the ladder (AC) to be 24 feet. When the top of the ladder slips 4 feet, the height of the ladder (RC) is now 20 feet. By again applying the Pythagorean theorem, this time to ΔPRC, we find PC = 15. (Note: ΔPRC is similar to a 3, 4, 5 right triangle; hence its sides are multiples of 3, 4, and 5.) PB = PC - BC = 15 - 7 = 8.



19. (C)



Let x = the length of the altitude to the side of length 14.
y = BD.
By applying the Pythagorean theorem:

to ΔADB: $x^2 + y^2 = 169$

$x^2 = 169 - y^2$ (1)

to ΔADC: $x^2 + (14 - y)^2 = 225$

$x^2 = -y^2 + 28y + 29$ (2)

Equate lines (1) and (2):

$169 - y^2 = -y^2 + 28y + 29$
$28y = 140; \quad y = 5$
Hence $x = 12$.

20. (D) Let t = the length of the fish's tail.
Let b = the length of the fish's body.

$$t = 7 + \frac{1}{2}b$$

$$b = 7 + t$$

Substitute for b in the first equation:

$$t = 7 + 2(7 + t)$$

$$t = 21$$

Therefore, b = 28.
Hence the length of the entire fish = h + b + t = 7 + 28 + 21 = 56.

TEST 15 SOLUTIONS

1. (C) Subtract: $\frac{34}{48} - \frac{5}{8} = \frac{17}{24} - \frac{15}{24} = \frac{1}{12}$. Hence x = 12.

Alternate Solution:
Let x = the number to be found

$$\frac{5}{8} + \frac{1}{x} = \frac{17}{24}$$

Multiply both sides of the equation by 24x (the lowest common denominator):

$$5(3x) + 24 = 17x;$$

$$24 = 2x;$$

$$x = 12.$$

2. (E) The correct order is $-\frac{1}{2} < \frac{1}{2} < .75$.

3. (E) Let w = the weight of the brick. $w = 5 + \frac{1}{2}(w)$; $w = 10$

4. (B) Since the profit is $\frac{1}{5}$ of the selling price, the cost is $\frac{4}{5}$ of the selling price. We may set up the following proportion: $\frac{\text{profit}}{\text{cost}} = \frac{\frac{1}{5}\text{ selling price}}{\frac{4}{5}\text{ selling price}}$; $\frac{\text{profit}}{\text{cost}} = \frac{1}{4} = 25\%$.

5. (A) The average spent per stamp by Ernie $= \frac{4(100) + 5(100)}{200} = 4\frac{1}{2}$ ¢. The average spent per stamp by Al $= \frac{4(500) + 5(500)}{1000} = 4\frac{1}{2}$ ¢. Hence the difference between these averages is zero.

6. (D) $\frac{1}{x} = \frac{1}{a} + \frac{1}{b}$

$$\frac{1}{x} = \frac{b + a}{ab} \quad \text{(adding fractions)}$$

$$\frac{x}{1} = \frac{ab}{b + a} \quad \text{(inverting both sides of the equation)}$$

$$x = \frac{ab}{a + b}$$

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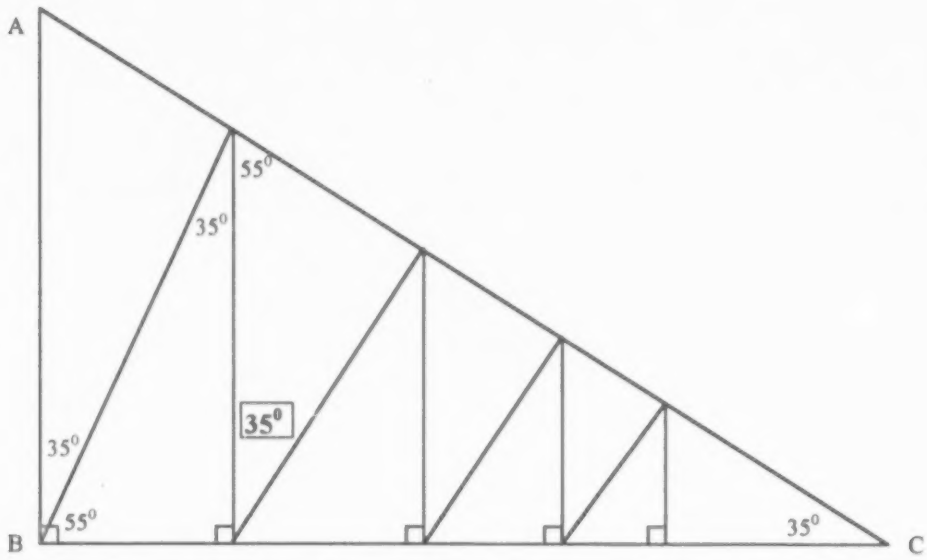
7. (B)



The angles in the diagram above were obtained by observing that the acute angles of a right triangle are complementary (i.e., have a sum of 90^0).

8. (C) This sequence (known as the Fibonacci series) has the following pattern:

$0 + 1 = \mathbf{1}$
$1 + 1 = \mathbf{2}$
$1 + 2 = \mathbf{3}$
$2 + 3 = \mathbf{5}$
$3 + 5 = \mathbf{8}$
$5 + 8 = \mathbf{13}$
$8 + 13 = \mathbf{21}$
$13 + 21 = \mathbf{34}$
$21 + 34 = \mathbf{55}$
etc.

9. (B) The new size = $\frac{3}{4}$ of the old size. The new weight = $\frac{1}{2}$ of the old weight. Hence the new $\frac{\text{weight}}{\text{size}} = \left(\frac{\frac{1}{2}}{\frac{3}{4}}\right)$ of the old size = $\frac{2}{3}$ of the old size.

10. (B) Let s = the selling price.

Since $62\frac{1}{2}\% = \frac{5}{8}$, $\frac{5}{8}$ s = the cost; $\frac{1}{3}$ s = the sales expenses.

The selling price = cost + sales expenses + profit. Therefore, $s = \frac{5}{8}s + \frac{1}{3}s + 1.50$.

Multiply both sides of the equation by 24 (the lowest common denominator):

$s = 36$
$\frac{5}{8}s = \$22.50$

11. (D) Let c = the cost of the boat. The expected sale price = c + (.10)(c). Therefore, (1.1)(c) - 50 = c - 15; c = \$350. However, since Bert sold the boat for \$15 less than the cost, the actual selling price was \$350 - \$15 = \$335.

12. (D) For the first four laps, the car travels a total time of (4)(50) or 200 seconds (i.e., 50 seconds each lap). For the next six laps (at the rate of 60 seconds each lap), the car travels a total time of (6)(60) or 360 seconds. Hence the total time for all 10 laps is 200 + 360 = 560 seconds. Therefore, the average time per lap for all 10 laps is $\frac{560 \text{ seconds}}{10 \text{ laps}}$ = 56 seconds per lap.

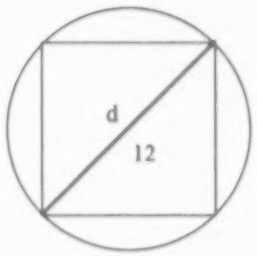
13. (A) The area of the shaded region is found by subtracting the area of the two small semicircles from the area of the large semicircle. Hence the area of the shaded region = $18\pi - 2\left(\frac{9}{2}\pi\right) = 9\pi$.

14. (C) The diagonal of the square is equal to the diameter of the circle. Since the area of a square = $\frac{1}{2}$ (diagonal)^2:

$72 = \frac{1}{2} d^2$; d = 12 (d = the diagonal = the diameter).

Since the radius of the circle is 6, the area = 36π

(Note: The diagonal of the square may also be found by using the Pythagorean theorem in one of the isosceles right triangles, with leg of length $\sqrt{72}$.



15. (B) Let H = Henry's Age.

H - 3 = Evelyn's Age

2H - 3 = The sum of their ages

Let us consider each of the two conditions separately:

First: $2H - 3 > 30$

$2H > 33$

$H > 16\frac{1}{2}$

Therefore, Henry must be age 17 or more.

Second: $2H - 3 < 35$

$2H < 38$

$H < 19$

Therefore, Henry must be age 18 or less. By combining both conditions (i.e., the intersection of the sets), we find that Henry must be either 17 or 18.

16. (C) We seek $\frac{50}{360}$ of the circumference; hence $\frac{5}{36}(\pi D) = \frac{5}{36}\left(\frac{22}{7}\right)(3) = \frac{55}{42}$ ("cancel" before multiplying).

17. (A) Since one boy won 3 times, the other boy had to win 3 times to "break even" and then win another 7 times to be ahead by 7 pennies. In all, the boys matched pennies 3 + 3 + 7 or 13 times.

18. (C) Since the total number of votes is 45, and A and E together have 28 votes, the votes of B + C + D = 17. If B has 6 votes, then C or D (not both) would have at least 6 votes. However, B must have more votes than C or D. Therefore, B must have more than 6 votes. Thus the smallest number of votes that B could have received is 7.

19. (C) Since DE || BC, $\angle AED = \angle ACB = 40^0$. $\angle AEF$ is complementary to $\angle AED$; hence it is equal to 50^0. $\angle FEC$ is supplementary to $\angle AEF$; hence it is equal to 130^0.

20. (D) Since $\frac{3}{2}$ hens work for $\frac{3}{2}$ days, we may speak of the job as $\left[\frac{3}{2}\right]\left[\frac{3}{2}\right]$ or $\frac{9}{4}$ "hen-days." Similarly, the second job is (6)(8) or 48 "hen-days." Thus we form the following proportion. Let x = the number of eggs laid by 6 hens in 8 days.

$$\frac{\frac{9}{4}\text{ hen - days}}{48\text{ hen - days}} = \frac{\frac{3}{2}\text{ eggs}}{\text{x eggs}}$$

Cross multiply: $\frac{9}{4}x = (48)(\frac{3}{2}) = 72$; $\quad x = 32$.

As an **alternate solution**, we may set up the following table:

$\frac{3}{2}$ hens lay $\frac{3}{2}$ eggs in $\frac{3}{2}$ days.

3 hens lay 3 eggs in $\frac{3}{2}$ days.

3 hens lay 6 eggs in 3 days.

3 hens lay 2 eggs in 1 day.

6 hens lay 4 eggs in 1 day.

6 hens lay **32 eggs** in 8 days.

TEST 16 SOLUTIONS

1. (C) Let x = the number of pencils which may be bought for S cents. Form the following proportion:

$$\frac{100\text{ pencils}}{\text{x pencils}} = \frac{\text{C cents}}{\text{S cents}} \quad \text{Cross multiply: } Cx = 100S; \quad x = \frac{100S}{C}$$

2. (A) Let x = the number of pounds that the 10-inch bar weighs. Form the following proportion (remember that 5 feet = 60 inches): $\frac{60\text{ inches}}{10\text{ inches}} = \frac{9\text{ pounds}}{\text{x pounds}}$; $\quad x = 1.5$.

3. (B) 550 yards = $\frac{550}{1760}$ miles

3 seconds = $\frac{3}{3600}$ hours

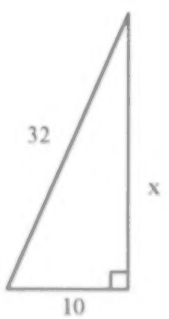
The rate of the airplane is (distance) ÷ (time): $\frac{550}{1760} \div \frac{3}{3600} = 375$ miles per hour.

4. (C) The set of numbers containing elements which are in both sets at the same time (i.e., the intersection of the sets) is {1, 2, 3, 4, 9, 12}, which may also be written as: Set A - { 5, 8, 10, 16}.

5. (E) Multiply both sides of the equation by 3:

$$3 = 1 - \frac{x}{y}; \quad \frac{x}{y} = -5; \quad x = -5y.$$

6. (C) Use the Pythagorean theorem to find x: $x^2 + 10^2 = 32^2$; $x = \sqrt{924} \approx 30$ {since 924 is closer to 900 [i.e., $(30)^2$] than to 961 [i.e., $(31)^2$].



7. **(C)** Since the height of the triangle is 4 and the base is 7, the area is $\frac{1}{2}(4)(7) = 14$.

8. **(D)** Since TIME = $\frac{\text{DISTANCE}}{\text{RATE}}$, the time consumed at the rate of 25 miles per hour = $\frac{175}{25} = 7$ hours. The time consumed at the rate of 35 miles per hour = $\frac{175}{35} = 5$ hours. Thus Judy would have saved 2 hours by traveling 10 miles per hour faster.

9. **(C)** since $8 * 8 = \frac{8 \cdot 8}{8 + 8} = 4$, $8 * (8 * 8) = 8 * 4 = \frac{8 \cdot 4}{8 + 4} = 2\frac{2}{3}$.

10. **(A)** The altitude of the rectangle is the same as the altitude of the triangle, and its base is 3 times as long. The area of the triangle = $\frac{1}{2}$ bh = 21; bh = 42. In the rectangle, the height is the same and the base is 3 times as large, so the area of the rectangle = (3b)(h) = 3(bh) = 3(42) = 126.

11. **(B)** Ordinarily, Al would pay \$1.20 for the 12 cans; however, during the sale he pays \$.31 for 4 cans, or \$.93 for 12 cans, a savings of \$.27.

12. **(A)** Let W = the number of votes the winner received.
(W - 22), (W - 30), and (W - 73) represent the number of votes his opponents received, respectively.

Therefore:

$$W + (W - 22) + (W - 30) + (W - 73) = 5219$$
$$4W - 125 = 5219$$
$$W = 1336$$

13. **(E)** Let C = the cost of the cork in cents.
105 - C = the cost of the bottle in cents.
Since the bottle costs 100 cents more than the cork,
$105 - C = C + 100$
$C = 2\frac{1}{2}$ cents

14. **(C)** $C = \frac{5}{9}(F - 32)$

Multiply both sides of the equation by $\frac{9}{5}$:

$\frac{9}{5}C = F - 32$

$F = \frac{9}{5}C + 32$

(This equation is a conversion formula in the reverse direction: it converts C units to F units.)

Thus, if $C = 20^0$, then $F = \frac{9}{5}(20) + 32 = 68^0$.

15. **(E)** Let *l*, *w*, and *h* be the three dimensions of the box.
$lw = 108 = (12)(9)$
$lh = 84 = (12)(7)$
$wh = 63 = (9)(7)$
After arranging the above so that *only three different factors* are used to represent the given areas, we may conclude that the dimensions of the box are: 12", 9", and 7".

16. (B) Contained in ΔDEF are four congruent triangles; therefore, when we subtract ΔABC from ΔDEF, we are left with an area (comprised of the three triangles DAB, AEC, and BCF) which is equal to three times the area of ΔABC. Hence the area of $\Delta ABC = \frac{1}{3}(24) = 8$.

17. (C) The number of peanuts is (12)(20) or 240. Divided by 15 children, this number provides 16 peanuts per child.

18. (B) Let x = the amount of acid to be added. Since there are 8 gallons (20% of 40) of acid in the original solution, 8 + x represents the amount of acid in the new solution; 40 + x represents the total amount of contents in the new solution.

$$\frac{8 + x}{40 + x} = \frac{60}{100} = \frac{3}{5}$$

Cross multiply: $5(8 + x) = 3(40 + x)$; $x = 40$ gallons.

19. (A) Let us assume that there were only 4 students in the class. Since $\frac{3}{4}$ (i.e., 75%) of the class passed, 3 people passed. Their average was 80 in this case, or we can assume that each of them received a grade of 80. What we seek is the failing mark.

Let x = the failing mark.

The entire class average = $70 = \frac{80 + 80 + 80 + x}{4}$.

Therefore, x = 40.

(Note: Many times we may choose a specific case of a problem (as above) without sacrificing the "generality" inherent in the problem. The technique of employing a specific case is very useful because it sometimes makes a problem more intelligible.)

20. (E) Let x = the number of soldiers in the army. $\frac{1}{20}x$ = the number of soldiers in the army who were *killed*.

$\frac{19}{20}x$ = the number of soldiers in the army who were *not killed*.

$\left(\frac{1}{8}\right) \bullet \left(\frac{19}{20}x\right)$ the number of soldiers in the army who were wounded.

$\left(\frac{1}{6}\right) \bullet \left(\frac{1}{8}\right) \bullet \left(\frac{19}{20}x\right)$ = the number of soldiers in the army who were mortally wounded.

$$\frac{1}{20}x = \left(\frac{1}{6}\right) \bullet \left(\frac{1}{8}\right) \bullet \left(\frac{19}{20}x\right) + 290$$

Multiply both sides of the equation by the lowest common denominator (i.e., 6 • 8 • 20):

$(8)(6)x = 19x + (6)(8)(20)(290)$;
$29x = (6)(8)(200)(29)$;
$x = 9{,}600$

TEST 17 SOLUTIONS

1. (C) There are several ways to approach this problem.

Solution 1: Raise each quantity to the 15th power (the lowest common denominator of the two exponents):

$$\left(8^{\frac{2}{3}}\right)^{15} = 8^{10} = \left(2^3\right)^{10} = 2^{30}$$

$$\left(32^{\frac{2}{5}}\right)^{15} = 32^6 = \left(2^5\right)^6 = 2^{30}$$

Therefore, the two original quantities are equal.

Solution 2:

$$8^{\frac{2}{3}} = \left(8^{\frac{1}{3}}\right)^2 = 2^2 = 4$$

$$32^{\frac{2}{5}} = \left(32^{\frac{1}{5}}\right)^2 = 2^2 = 4.$$

Again we see that the two original quantities must be equal.

Solution 3: Since $8 = 2^3$, we have $8^{\frac{2}{3}} = \left(2^3\right)^{\frac{2}{3}} = 2^2 = 4$.

Similarly, $32 = 2^5$; therefore, $32^{\frac{2}{5}} = \left(2^5\right)^{\frac{2}{5}} = 2^2 = 4$.

2. (B) When a *negative* number is raised to an *odd* power the result is *negative*; and when a *negative* number is raised to an *even* power, the result is *positive*. Since x is negative ($x < 0$), x^3 is also negative and therefore $\frac{1}{x^3}$ is negative. Since x^2 is positive, $\frac{1}{x^2}$ is also positive.

Therefore $\frac{1}{x^2} > \frac{1}{x^3}$

3. (A) To solve the given inequality, divide both sides by -5 to get $x < -4$. (Reminder: When multiplying or dividing both sides of an inequality by the same *negative* number, the *order* of the inequality must be changed.) Since $x < 4$ and $-4 < -3$, we have $x < -3$.

4. (B)

[-1.5] is -2
[-1] is -1
$-1 > -2$

5. (D) From the fact that $\frac{a}{b} > 0$ we can conclude nothing about the sizes of a and b, since both might be negative or both positive .

6. (A) Since the sum of the lengths of any two sides of a triangle is greater than the length of the third side, $a + b > c$.

7. (B) Multiply both sides of $s > t$ by -1 to get $-s < -t$. Adding $x = y$ to that inequality then yields $x - s < y - t$.

8. (A) Solution 1: Since $n < 0$ and $y < x$, we have $ny > nx$. Dividing both sides by the positive number yx, we get $\frac{n}{x} > \frac{n}{y}$.

Solution 2: Since $x > y > 0$, we have $\frac{1}{x} < \frac{1}{y}$. Multiplying by the negative number n yields $\frac{n}{x} > \frac{n}{y}$.

9. (A) If 1 meter = 1.1 yards, then 2.2 meters = (2.2)(1.1) yards = 2.42 yards, which is greater than 2.2 yards.

10. (B) Since $\angle B = \angle C$ and $\angle B > 45^0$, we have $\angle B + \angle C > 90^0$. However, $\angle A + \angle B + \angle C = 180^0$. Therefore, $\angle A < 90^0$.

11. (A) If two sides of a triangle are of unequal length, then the angles opposite them are also of unequal measure; the angle of greater measure is opposite the longer side. Therefore, $\angle B > \angle C$.

12. (B) The easiest way to solve the equation is to divide both sides by 6 to get: $(x - 10)^2 = 25$. Then $x - 10 = \pm 5$, so that $x = 15$ or $x = 5$, each of which is less than 16.

13.(A) Since $m = n + 1$, we have $m - n = 1$ and $n - m = -1$.
Therefore, $(m - n)^5 = (1)5 = 1$, and $(n - m)^5 = (-1)^5 = -1$.
Hence $(m - n)^5 > (n - m)^5$.

14. (A) Since $\sqrt{x^2} = |x|$, it follows that y is nonnegative and therefore greater than -1.

15. (A) Dividing both sides of the given inequality by the positive number bd yields $\frac{a}{b} > \frac{c}{d}$.

16. (B) When a negative number is raised to an *odd* power the result is negative, and when a *negative* number is raised to an *even* power the result is *positive*. Since $x = -1$, we have $x^5 + x^7 = (-1)^5 + (-1)^7 = -1 + (-1) = -2$, and $x^3 + x^4 = (-1)^3 + (-1)^4 = -1 + 1 = 0$, which is greater than -2.

17. (C) The average of an odd number of consecutive terms in an arithmetic progression is the middle term. Therefore, $x + 1$ (the middle term) is the average in each case. Although more time-consuming, the usual method for computing an average may also be used.

For column A: $\frac{3x + 3}{3} = x + 1$ For column B: $\frac{5x + 5}{5} = x + 1$

18. (B) Since the sum of the lengths of any two sides of a triangle is greater than the length of the third side, $b + c > a$. Subtracting b from both sides of the inequality yields $c > a - b$.

19. (D) If $d = 1$, then $10d + 1 = 11$, and $d + 10 = 11$. However, if $d = 2$, then $10d + 1 = 21$, and $d + 10 = 12$. Therefore, it is necessary to know the value of d to determine the relative size of the two quantities.

20. (C) If the number in column A is multiplied by $\frac{\sqrt{5}}{\sqrt{5}}$, which equals 1, we get $-\sqrt{5}$, the number in column B.

21. (A) At 11:45 the minute hand is on the 9 and the hour hand is *not* on the 11. If it were on the 11, the angle formed would be 60^0 (there are 30^0 between each of the 12 numeral intervals on the clock). The hour hand is actually located $\frac{3}{4}$ the distance from the 11 to the 12. Hence the acute angle at 11:45 is *greater* than 60^0 [exactly $\frac{3}{4}(30)+60 = 82\frac{1}{2}$].

At 3:25 the minute hand is on the 5 and the hour hand is *not* on the 3. If it were on the 3, the angle formed would be 60^0. The hour hand is actually located $\frac{5}{12}$ (i.e. $\frac{25}{60}$) the distance from the 3 to the 4. Hence the acute angle at 3:25 is *less* than 60^0 [exactly $\frac{7}{12}(30)+30 = 47\frac{1}{2}$].

22. (B) Since the opposite angles of a quadrilateral inscribed in a circle are supplementary, $3x + 70 = 180$. Therefore, $x = 36\frac{2}{3}$, which is greater than 36.

23. (C) Parallel lines are everywhere equidistant. Therefore, the altitude from B of ΔABC is the same length as the altitude from D of ΔADC. Both ΔABC and ΔADC have a common base, AC. Thus their areas are equal.
Recall: Area of a triangle = ($\frac{1}{2}$)(base)(altitude).

24. (B) Since the sum of the lengths of any two sides of a triangle is greater than the length of the third side, $y + y > 3x$, or $2y > 3x$.
Dividing both sides of the inequality by 2 yields $y > \frac{3}{2}x$, which implies that $y > x$ (since $\frac{3}{2}x > x$).

25. (B) Solution 1: The measure of an exterior angle of a triangle is greater than the measure of each remote interior angle. So for ΔAED, $\angle BDC > \angle AED$, and for ΔAEB, $\angle AED > \angle BAE$. By transitivity, $\angle BDC > \angle BAE$.
Solution 2: Since $\angle BDC$ is an exterior angle of ΔBAD, $\angle BDC > \angle BAD$. However, $\angle BAD > \angle BAE$. By transitivity, $\angle BDC > \angle BAE$.

26. (D) Since, for example, $|-2| = |2|$ and $|3| = |-3|$, it is impossible to decide whether x or y is greater. (From $|x| = |y|$ we can only conclude that either $x = y$ or $x = -y$.)

27. (B) To find the average speed over the same distance, find the harmonic mean of the speeds:

$$\frac{2R_1R_2}{R_1 + R_2} = \frac{(2)(40)(60)}{40 + 60} = 48,$$ which is less than 50.

28. (C) Solution 1: The sum of the measure of the angle formed by the two tangents (CA and CB) and the measure of the closer intercepted arc AB is 180^0. So $AB = 90^0$.
Solution 2: Draw AO and BO. Since $AO \perp AC$ and $BO \perp BC$, $\angle AOB$ must be 90^0 (since the sum of the measures of the angles of a quadrilateral is 360^0). Therefore, $AB = 90^0$.
Solution 3: $\angle ACB = \frac{1}{2}[\overset{\frown}{ADB} - \overset{\frown}{AB}]$. Hence $90 = \frac{1}{2}[(360 - AB) - \overset{\frown}{AB}]$, and so $90 = 180 - AB$.
Therefore, $\overset{\frown}{AB} = 90^0$.

29. (A) The easiest way to solve this problem is to compare the cross products $x(y + k)$ and $y(x + k)$, which equal $xy + xk$ and $xy + yk$, respectively. Since k is positive and $x > y$, we have $xk > yk$; so the first cross-product is the greater. Therefore, $\frac{x}{y} > \frac{x + k}{y + k}$.

30. (A) If two chords of a circle do not have the same length, then the longer chord is nearer the center of the circle. Therefore, OB > OA.

31. (C) Two tangent segments to a circle from the same exterior point have the same length.
Therefore, a = b, c = d, and e = f. Hence a + c + e = b + d + f.

32. (B) If $\angle B = 90^0$, then $(AC)^2 = (AB)^2 + (BC)^2$ by the Pythagorean theorem. Suppose that the angle is hinged at B and increases to 130^0, with AB and BC remaining constant. Then AC would have to increase.
Hence $(AC)^2 > (AB)^2 + (BC)^2$.

33. (D) If one of x or y is negative and the other is positive, then $|x + y| < |x| + |y|$. If both x and y are positive or both are negative, then $|x + y| = |x| + |y|$. A comparison of the two quantities is therefore not possible without further information.

TEST 18 SOLUTIONS

1. (A) When a product (in this case 7x) equals 0, at least one factor must be 0.
Therefore, x = 0, which is less than 1.

2. (B) When multiplying numbers, an *odd* number of negative numbers produces a *negative* result, whereas an even number of negative numbers produces a positive result. Therefore the quantity in column A is negative (three negative numbers), whereas the quantity in column B is positive (four negative numbers). A positive number is greater than every negative number.

3. (A) If both sides of the inequality are multiplied by the positive number y, the result is $x > y$.

4. (A) Since $\angle C = 90^0$, $\angle C$ is the angle of greatest measure in the triangle. The longest side of a triangle is opposite the angle of greatest measure. Therefore, AB (opposite $\angle C$) is the longest side; in particular, AB > AC.

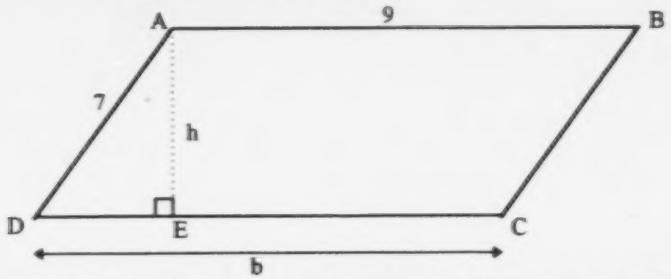
5. (B) Since 1 meter = 3.3 feet, 1.1 meters = (1.1)(3.3) feet = 3.63 feet, which is less than 4.4 feet.

6. (A) Increasing the denominator of a fraction with a positive numerator decreases the fraction.
Therefore, $\frac{m}{17} > \frac{m}{19}$.

7. (B) An exterior angle ($\angle ACD$) of a triangle is greater than each remote interior angle.
Therefore, $\angle ACD > \angle A$.

8. (D) The answer must be D, since 2 examples of n, with 2 different answers can easily be found to demonstrate the ambiguity. If n = 2, and n + 1 = 3, each has 2 divisors.
If n = 24 and n + 1 = 25, 24 has 8 divisors (1, 2, 3, 4, 6, 8, 12, 24), whereas 25 has 3 divisors (1, 5, 25).

9. (B) The area of a nonrectangular parallelogram equals bh (see figure).
Since AD is the hypotenuse of right ΔADE,



AE (or h) < 7. Therefore, the area of parallelogram ABCD = bh < 9 X 7 = 63 square inches (note: b = 9).
However, the area of a rectangle with sides of lengths 7 inches and 9 inches equals 7 X 9 = 63 square inches.
Thus the area of the rectangle is greater than the area of the non rectangular parallelogram.

10. (B) To solve the inequality $7 - 12x > 5$, add $12x - 5$ to both sides to get $2 > 12x$.

Dividing both sides by 12 yields $\frac{1}{6} > x$. Thus $x < 2$.

11. (D) If $x = y$, then $(x - y)^3 = 0 = (y - x)^3$. If $x \neq y$, then $(x - y)^3$ and $(y - x)^3$ are unequal, one being positive and the other negative. Hence a comparison cannot be made without further information.

12. (D) Nothing is known here about the relative magnitudes of x and y.

13. (B) The two marked angles in the diagram are supplementary; therefore, $5x + (180-4y) = 180$.

Then $5x = 4y$, and so $x = \frac{4}{5} y$. Hence $x < y$.

14. (D) $(x + 8)(x - 3) = x^2 + 5x - 24$ and $(x + 7)(x - 4) = x^2 + 3x - 28$. So we must compare $5x - 24$ with $3x - 28$. But if $x = -2$, then those two quantities are equal, whereas they are unequal if $x \neq -2$. Hence no comparison can be made without further information.

15. (A) Since $a > b$ and c is negative, we have $ac < bc$. Dividing both sides by the positive number ab,

we get $\frac{c}{b} < \frac{c}{a}$.

16. (C) Multiplication in each column reveals that both quantities equal xy - xz.

17. (A) Since $x = -2$, we have $\frac{x^5}{x^3} = x^2 = (-2)^2 = 4$, and $\frac{x^7}{x^4} = x^3 = (-2)^3 = -8$. But $4 > -8$.

Hence the number in column A is the greater.

18. (A) Both triangles ABD and ADC have the same altitude from A to BC. Therefore, the triangle with the longer base has the greater area. Since BD > DC, ΔABD has the greater area.

19. (A) $(x + y)^2 = x^2 + 2xy + y^2$. Since $x^2 + y^2 = 8$ and $2xy = 6$, it follows that $(x + y)^2 = 14$, which is greater than 8.

20. (C) The measure of an angle formed by a tangent and a secant drawn from a point outside a circle is equal to one half the difference of the measures of the two intercepted arcs; that is, $x = \frac{1}{2}(a - b)$. Therefore, $2x = a - b$.

21. (C) If you multiply the numerator and denominator of the number in column A by $\sqrt{2} + 1$, you get the number in column B. Therefore, the numbers are equal. A calculator would be useful here.

22. (A) An exterior angle of a triangle has greater measure than each remote interior angle. So in ΔDAC we have $a > b$. But $b = c$. Therefore, $a > c$.

23. (B) Since both denominators are positive, it suffices to compare the cross products $a(b - m)$ and $b(a - m)$, or $ab - am$ and $ba - bm$. Hence we need only compare $-am$ and $-bm$. Since $a < b$ and $-m$ is positive, we have $-am < -bm$. Therefore, the second cross product is the greater and so the quantity in column B is the greater.

24. (C) Since the opposite angles of a quadrilateral inscribed in a circle are supplementary, $\angle A + \angle C = 180^0 = \angle B + \angle D$.

25. (B) The surface area of a cube of edge length e is $6e^2$ (the sum of the areas of the six square faces). The volume of this cube is e^3. Since $6e^2 > e^3$, then $6 > e$.

26. (D) Although the two triangles have the same altitude (not necessarily AD however), we know nothing about the relative sizes of their bases, BD and DC. Hence no comparison of the areas can be made.

27. (D) When $m = 1$, the remainder, 2, of $\frac{56}{3m}$ is greater than the remainder, 1, of $\frac{58}{3m}$. However, when $m = 2$, the remainder, 2, of $\frac{56}{3m}$ is less than the remainder, 4, of $\frac{58}{3m}$. Hence no comparison can be made without further information.

28. (A) Imagine that AB, BD, and BC are sticks joined at a hinge and AD and DC are rubber strips. Clearly, the larger the measure of $\angle ABD$, the longer AD will be. This intuitively justifies the conclusion that $AD > DC$ (since $\angle ABD > \angle CBD$).

29. (C) From an external point of a circle two tangent segments to the circle have the same length. Therefore, $AE = AH$, $BE = BF$, $CG = CF$, and $DG = DH$. By addition, $AE + BE + CG + DG = AH + DH + BF + CF$; so $AB + DC = AD + BC$.

30. (A) The measure of the acute angle formed by the hands of a clock at 2:00 is 60^0. But the measure of the acute angle formed by the hands at 2:20 is not 60^0 (the degree measure between the 2 and the 4) because the hour hand is *not* on the 2. Indeed, the hour hand has moved one third the distance from the 2 to the 3, so that the measure of the angle is $\frac{2}{3}(30) + 30$, or 50^0.

31. (C) Since $AD = DC$, $\angle A = x^0$; and since $CD = DB$, $\angle DCB = y^0$. Then the sum of the measures of the angles of $\Delta ABC = 2x^0 + 2y^0 = 180^0$, and so $x^0 + y^0 = 90^0$. But $\angle ACB = x^0 + y^0$. Hence $\angle ACB = 90^0$. An alternate solution may be given as follows: Since $DA = DC = DB$, a circle with center D may be passed through points A, C, and B. Since $\angle ACB$ is inscribed in a semicircle, $\angle ACB = 90^0$.

32. (A) Draw $RH \perp THN$. Since $\angle PQR = 90$, $\angle RQN$ is complementary to $\angle PQT$. Also, $\angle TPQ$ is complementary to $\angle PQT$. Therefore, $\angle RQN = \angle TPQ$, and $PQ = QR$. Thus $\Delta PTQ \cong \Delta QHR$ (A.A.S.) and $QH = PT = 4$. Then $TH = 7$. Since TR is the hypotenuse of ΔTHR, TR is the longest side of the triangle. Thus $TR > TH$, or $TR > 7$.

33. (B) $\angle PRQ = 180^0 - 60^0 - 61^0 = 59^0$. Therefore, $\angle PRQ < \angle PQR$. So $PQ < PR$ (in a triangle, the longer of two sides is opposite the greater angle). The area of a triangle $= \frac{1}{2}$ bh. Hence the area of ΔPQR equals both $\frac{1}{2}(RM)(PQ)$ and $\frac{1}{2}(QN)(PR)$. So $(RM)(PQ) = (QN)(PR)$. But we noted above that $PQ < PR$. Therefore, $RM > QN$.

TEST 19 SOLUTIONS

1. (B) The pattern in the first sequence is

$2 + 3 = 5$
$3 + 5 = 8$
$5 + 8 = 13$
$8 + 13 = \mathbf{21}$

The second pattern is

$1 \times 2 = 2$
$2 \times 4 = 8$
$4 \times 8 = \mathbf{32}$

2. (A) The sum of the lengths of any two sides of a triangle is greater than the length of the third side. Therefore, $a + 3 > 5$. Subtracting 3 from both sides of the inequality yields $a > 2$.

3. (B) Since $p < 0$, p is negative. When a negative number is raised to an *odd* power the result is negative. Therefore, p^3 is negative and so $p^3 - 1 < -1$.

4. (D) We examine the cross products. If $b(b - a) > (a - b)(a - b)$. Then the (A) fraction is the larger and so on. This yields $b^2 - ab \stackrel{?}{=} b^2 - ab + [a^2 - ab]$. The comparison depends on the value of $a^2 - ab$, which depends on the relation of a to b, which we do not know.

5. (A) Since AB = AC, we have $3x = 5y$. Dividing both sides by 3, we get $x = \frac{5}{3}y$, which implies that $x > y$.

6. (C) Since 1 kilogram = 2.2 pounds, we have 15 kilograms = (15)(2.2) pounds = 33 pounds.

7. (C) By adding we get: $\frac{1}{x} + \frac{1}{y} = \frac{y}{xy} + \frac{x}{xy} = \frac{y + x}{xy}$.

8. (B) When dividing (or multiplying) both sides of an inequality by the same *negative* number, the order of the inequality must be changed. Dividing both sides of $xm > ym$ by the negative quantity m, we get $x < y$.

9. (A) To solve this inequality, add $1 - 2x$ to both sides of the inequality to get $6 < 3x$. Dividing both sides by 3 yields $2 < x$.

10. (B) Multiply both sides of $\frac{a}{b} < 1$ by the positive number b to get $a < b$.

11. (B) Since $\frac{2}{x} = \frac{3}{y}$, we have $2y = 3x$ and so $y = \frac{3}{2}x$, which is greater than x (because x is positive).

12. (B) Evaluating each quantity for $x = -1$, we get $x^{-5} = (-1)^{-5}$
$= \frac{1}{(-1)^5} = \frac{1}{-1} = -1$, and $x^{-8} = (-1)^{-8} = \frac{1}{(-1)^8} = \frac{1}{1} = 1$.
(Note: An odd power of a negative number is negative, but an even power of a negative number is positive.)

13. (B) Dividing both sides of $a > b$ by the positive quantity ab yields $\frac{1}{b} > \frac{1}{a}$. Multiplying both sides of that inequality by the positive quantity c gives $\frac{c}{b} > \frac{c}{a}$.

14. (C) The average of $a - 2$, a, $a + 2$ is $\frac{3a}{3} = a = b + 3$. The average of $b - 2$, $b + 5$, $b + 6$ is $\frac{3b + 9}{3} = b + 3$.

15. (D) Although it is possible to solve the equation by squaring the binomial x - 7, multiplying by 5, and then solving the quadratic, that is *not* the quickest way to solve the equation. The best way is to divide both sides of the equation by 5 to get $(x - 7)^2 = 9$; then take the square root of both sides to get $x - 7 = \pm 3$, and so $x = 10$ or $x = 4$. Therefore, no comparison can be made.

16. (B) Since AB = AC, $\angle ABD = \angle ACB$. An exterior angle of a triangle is greater than each remote interior angle. Therefore, for ΔADC, $\angle ADB > \angle ACB$. Hence $\angle ADB > \angle ABD$.

17. (C) Both x^0 and y^0 equal ½ arc AB. Therefore, x = y.

18. (D) Since the integers n and m are clearly positive, the in equalities are equivalent to $11 < n^2 < 20$ and $3 < m^2 < 27$, respectively. The only solution for the first inequality is 4.
But the second inequality has solutions 2, 3, 4, and 5. Hence no comparison is possible.

19. (D) If ΔABC is isosceles, then h = m. But if ΔABC is not isosceles, then m > h, because the hypotenuse of a right triangle is the longest side. Hence no comparison can be made.

20. (C) Using the formula provided, we find:

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = (1)(4) - (2)(3) = -2$$

$$\begin{vmatrix} 4 & 3 \\ 2 & 1 \end{vmatrix} = (4)(1) - (3)(2) = -2$$

21. (A) Since each side of the square has length 9, the square's area is $9^2 = 81$. Each side of the equilateral triangle has length $\frac{36}{3} = 12$. Draw an altitude to form a $30^0 - 60^0 - 90^0$ triangle. By the Pythagorean theorem it follows that the altitude has length $6\sqrt{3}$. So the equilateral triangle has area $\frac{12 \bullet 6\sqrt{3}}{2} = 36\sqrt{3} \approx 62.4$, which is less than 81. (Note: You may wish to memorize a formula for the area of an equilateral triangle with side length s: $\frac{s^2\sqrt{3}}{4}$).

22. (A) The measure of the acute angle formed by the hands of a clock at 11:45 is not 60^0, since the hour hand is not on the 11. It has moved $\frac{3}{4}$ the distance from the 11 to the 12. The measure of the angle is therefore $\frac{3}{4}(30) + 60 = 82.5^0$. (Note: The degree measure between consecutive numbers on a clock is 30^0.)

The measure of the acute angle formed by the hands of a clock at 8:30 is not 60^0, since the hour hand is not on the 8. It has moved l/2 the distance from the 8 to the 9. The measure of the angle is therefore $\frac{1}{2}(30) + 60 = 75^0$.

23. (B) Since BD bisects $\angle ABC$ and $\angle ABC = 60^0$, $\angle ABD = 30^0$ and therefore, ΔABD is isosceles with AD = BD. In right ΔDCB, hypotenuse BD > DC. Therefore, AD > DC.

24. (C) The base of ΔAPC and of ΔBPC is PC. Both of these triangles also have equal altitudes (since AD = BC). Therefore, the triangles have equal areas.

25. (C) To find the area of the region shaded you merely find the area of the square and subtract the area of the circle. To find the area of the region shaded you find the area of the square and subtract the area of the *four quarter circles*. This is the same as subtracting the area of one entire circle (with the same radius). Since the areas of both shaded regions are found with the same computation, they are equal.

26. (D) Let $x = 2$ and $y = 1$. If $k = 0$, the two given fractions are equal. But if $k = 1$, the fractions are unequal. Hence no comparison can be made.

27. (D) The given inequality is true for $x = 8$ and for $x = 9$. Hence no comparison can be made.

28. (A) Let the list price be \$100. Since the discount is 8% of \$100, or \$8, the discounted price is \$92. Now add the sales tax 8% (\$92), or \$7.36, to get a final cost of \$99.36, which is less than the original list price.

29. (B) $\angle P \cong \frac{1}{2}(\overset{\frown}{TB} - \overset{\frown}{TC}) \cong \frac{1}{2}(\overset{\frown}{TB}) - \frac{1}{2}(\overset{\frown}{TC})$, and $\angle A \cong \frac{1}{2}(\overset{\frown}{TB})$. Therefore, $m\angle A > m\angle P$.

30. (B) Solution 1: First find the total number of gallons purchased: $\frac{700}{55} + \frac{700}{65} \approx 12.73 + 10.77 = 23.5$. The average price per gallon is the total price divided by the total gallons, or $1400 \div 23.5 \approx 59.6$ cents, which is less than 60 cents.

Solution 2: Find the harmonic mean of the rates 55 and 65: $\frac{(2)(55)(65)}{55 + 65} = \frac{715}{12} \approx 59.6$ cents, which is less than 60 cents.

31. (B) Since $x \neq y$, $(x - y)^2 > 0$. Therefore, $x^2 - 2xy + y^2 > 0$. By adding $4xy$ to both sides of the inequality we get:

$$x^2 + 2xy + y^2 > 4xy$$
$$(x + y)^2 > 4xy$$

Taking the square root of both sides (everything is positive), we get:

$$x + y > 2\sqrt{xy}$$
$$\frac{x + y}{2} > \sqrt{xy}$$

32. (B) Let h be the length of the altitude of the equilateral triangle. Then $h < JK$, because the hypotenuse of a right triangle is always the longest side. But $JK = LK = SR = QR$. Hence $h < QR$.

33. (A) It can be proved that for any given perimeter, the rectangle of greatest area is a square. You can test this as follows. Let the perimeter be 100 (any number divisible by 4 would be convenient). Then the square has area $25 \times 25 = 625$. Now try a rectangle with dimensions 24, 24, 26, 26 (for a total of 100). The area is $24 \times 26 = 624$, which is less than 625.

TEST 20 SOLUTIONS

1. (D) No comparison can be made. For example, if $a = 1$, $b = 0$, $c = -1$, then $a + c$ equals $2b$; but if $a = 3$, $b = 1$, $c = 0$, then $a + c$ and $2b$ are unequal.

2. (A) Since the product of an *odd* number of negative numbers results in a *negative* number, x is negative (the product of five negatives). Therefore, $x < 0$.

3. (A) The hypotenuse of a right triangle is the longest side of the triangle (since it is opposite the angle of greatest measure of the triangle). Therefore, in ΔBCD, $BD > DC$.

4. (B) Solution 1: Since $x^2 + 6$ is positive for every real value of x, it follows that 5x must be negative (otherwise the sum $x^2 + 5x + 6$ would not be 0). Therefore, x is negative and so $x < 0$.
Solution 2: Factor the left side of the equation and set each factor equal to 0:

$$(x + 2)(x + 3) = 0$$
$$x + 2 = 0 \mid x + 3 = 0$$
$$x = -2 \mid x = -3$$

In each case we have $x < 0$.

5. (D) If $n < 0$ then $\frac{|n|}{n} = -1$. If $n > 0$ then $\frac{|n|}{n} = 1$.

Therefore, we do not know enough about n to determine the value of $\frac{|n|}{n}$.

6. (D) No comparison can be made because y could be positive (e.g., $\frac{14}{2} = 7$) and y could be negative (e.g., $\frac{-14}{-2} = 7$).

7. (D) Since $x^2 = 1$, then $x = 1$ or $x = -1$. Similarly, since $y^2 = 9$, then $y = 3$, or $y = -3$. It is therefore impossible to compare x and y.

8. (C) From the given information, $ab = \left(\frac{2}{3}\right)\left(\frac{3}{2}\right) n^2 = n^2$.

9. (B) For ΔABD and ΔABC we know that $AB = AB$ and $AD = BC$. Hence we need only compare BD with AC. But clearly $BD < AC$, since the diagonals of a rhombus only have equal length if it's a square. So ΔABD has the smaller perimeter.

10. (A) First simplify each fraction: The first equals $\frac{x}{y}$, and the second equals $\frac{y}{x}$. Since $x > y > 0$, we have $x^2 > y^2$. Then dividing both sides by the positive quantity xy, we get $\frac{x}{y} > \frac{y}{x}$.

11. (A) $(\sqrt{3})^2 = 3$, and $(\sqrt{3} - 1)(\sqrt{3} + 1) = (\sqrt{3})^2 - 1^2 = 3 - 1 = 2$.

12. (B) Since the sum of the measures of the angles of a triangle is 180^0, $3x + 2x - 11 + 78 = 180$.
Therefore, $x = 22\frac{3}{5} > 18$.

13. (A) Since $a > b > 0$ and $a > c > 0$, we may multiply in equalities to get $a^2 > bc$. Then dividing both sides by the positive quantity ab, we have $\frac{a}{b} > \frac{c}{a}$.

14. (D) No comparison is possible because the two quantities could be equal (e.g., when $y = z = 0$) and could be unequal (e.g., when $x = y = z = 1$).

15. (C) Since all quantities are positive, we need only compare the cross products $a(b + d)$ and $b(a + c)$, which equal $ab + ad$ and $ab + bc$, respectively. But $ad = bc$ [because $\frac{a}{b} = \frac{c}{d}$]. So the cross products are equal and hence the two fractions are equal.

16. (D) The equation is satisfied by $x = 0$ and $y = \frac{4}{5}$ (in which case $x < y$) and is satisfied by $y = 0$ and $x = \frac{4}{9}$ (in which case $y < x$). Hence no comparison is possible.

17. (D) No comparison can be made because $x > y$ if k is positive, but $x < y$ if k is negative.
(Remember that the order of an inequality must be changed when multiplying or dividing by a negative quantity.)

18. (D) Since AB = AC, we know that angles B and C have equal measure. From that information, however, no comparison of angles A and B can be made. (Although $\angle B$ must be acute, $\angle A$ might be obtuse or acute.)

19. (C) Let AB = BC = d. Then the lengths of semicircular arcs $\overset{\frown}{AQB}$ and $\overset{\frown}{BRC}$ are each $\frac{1}{2}\pi d$: Therefore, the quantity in column B equals πd. The length of semicircular arc $\overset{\frown}{APC}$ is $\frac{1}{2}(2\pi d) = \pi d$. Thus the quantities in columns A and B are equal.

Alternate Solution: Since the sum of the diameters of the two smaller semicircular arcs equals the diameter of the larger semicircular arc, the same relationship is true for the semicircular arcs.

20. (B) From the definition provided,

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = (1)(4) - (2)(3) = -2, \text{ and}$$

$$\begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix} = (3)(2) - (4)(1) = 2.$$

21. (A) An exterior angle of a triangle has greater measure than each remote interior angle. Hence $x > z$.
Since AC = BC, we have $z = y$. Therefore, $x > y$.

22. (D) No comparison can be made because $10n$ and n^2 are equal if $n = 10$, but they are unequal if $n = 11$.

23. (A) The measure of the acute angle formed by the hands of a clock at 2:00 is exactly 60^0. (Note: The measure of the angle between consecutive numbers is 30^0.) However, the measure of the acute angle formed by the hands of a clock at 4:30 is *not* 60^0. The hour hand is *not* on the 4 when the minute hand is on the 6. The hour hand has moved ½ the distance between the 4 and the 5. Therefore, the measure of this acute angle is $\frac{1}{2}(30) + 30 = 45^0$.

24. (B) The area is πr^2 and the circumference is $2\pi r$. Multiply both sides of $r < 2$ by the positive quantity πr to get $\pi r^2 < 2\pi r$.

25. (B) The area of the smaller circle (radius OA = 3) equals πr. The area of the shaded region equals the difference between the areas of the circles with radii OB and OA, respectively: $25\pi - 9\pi = 16\pi > 9\pi$.

26. (D) Since all quantities are positive, we need only compare the cross products $mn + mk$ and $nm + nk$.
But mk and nk cannot be compared without further information.

27. (B) The quantity in column A equals

$$2 + \frac{1}{2 + \frac{1}{2}} = 2 + \frac{1}{\frac{5}{2}} = 2 + \frac{2}{5} = 2\frac{2}{5},$$

which obviously is less than the quantity in column B.

28. (A) If $\angle B$ were a right angle, then by the Pythagorean theorem we would have $(AB)^2 + (BC)^2 = (AC)^2$. Imagine that the angle at B is hinged and decreases to 70^0, with AB and BC remaining constant. Clearly AC would have to decrease. Therefore, $(AB)^2 + (BC)^2 > (AC)^2$.

29. (B) Solution 1: The measure of an exterior angle of a triangle exceeds the measure of each remote interior angle. For ΔABD we have $y > \angle ADB$, and for ΔDBC we have $\angle ADB > x$. By transitivity we get $y > x$.
Solution 2: For ΔABC we have $y > \angle ABC$. But $\angle ABC > x$. Hence $y > x$.

30. (B) The sum of the lengths of two sides of a triangle exceeds the length of the third side. For ΔAPB, $AP + PB > AB$. For ΔBPC, $PB + PC > BC$. For ΔAPC, $AP + PC > AC$.
Adding the three inequalities yields $2(AP + PB + PC) > AB + BC + AC$.

31. (A) Since ΔABC is equilateral, $\angle A = 60^0$. But $\angle A = \frac{1}{2}(\overset{\frown}{BC}) > \frac{1}{2}(\overset{\frown}{BC}) - \frac{1}{2}(\overset{\frown}{DE}) = \frac{1}{2}(\overset{\frown}{BC} - \overset{\frown}{DE}) = \angle P$.
Hence $60^0 > \angle P$.

32. (A) The measure of an exterior angle of a triangle exceeds the measure of each remote interior angle. For ΔADC we have $\angle ADB > \angle C$. But $\angle C = \angle B$ (because $AB = AC$). Hence $\angle ADB > \angle B$. If two angles of a triangle have different measures, the sides opposite those angles have different lengths, the longer side being opposite the greater angle. Therefore, in ΔABD we may conclude that $AB > AD$.

33. (A) Since ΔADE and ΔDFE have the same base (DE), a comparison of their areas depends on their altitudes. The distance from A to DC is obviously greater than the distance from F to DC. Therefore, area $\Delta ADE >$ area ΔDFE.

TEST 21 SOLUTIONS

1. ($\frac{7}{11}$ **or .636)** The required fraction is $\frac{\text{area of remaining paper}}{\text{area of original paper}} = \frac{7 \bullet 8\frac{1}{2}}{8\frac{1}{2} \bullet 11} = \frac{7}{11}$.

Had a calculator been used, then the answer might have come up as $.6363\overline{36}$. This is a repeating decimal. To change it to fraction form let

$x = .6363\overline{36}$
$10x = 6.3636\overline{36}$
$1000x = 636.36\overline{36}$

Subtracting,

$990x = 630$

$x = \frac{7}{11}$ **or .636**

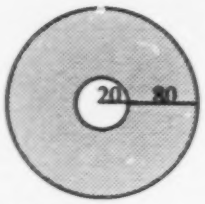
2. (6) It takes a job of $5 \times 4 \times 10 = 200$ worker hours to complete a job worth \$1200. Therefore, the cost per worker is $\frac{\$\,1{,}200}{200} = \$6$.

3. ($\frac{40}{3}$ **or** $13.\overline{3}$) Let x = the amount of wine sought.

$.12x = (.4)(4)$
$.12x = 1.6$

$x = \frac{160}{12} = \frac{40}{3}$ **or** $13.\overline{3}$

4. (96) After 20 revolutions, a large ring of width 4ft × 20 = 80ft. of the entire circle has been mowed. The fraction of the unmowed area is: $\frac{(20)^2\,\pi}{(100)^2\,\pi} = \left(\frac{1}{5}\right)^2 = \frac{1}{25} = \frac{4}{100}$ or 4%.
That makes the mowed percent 96.



5. (7) To make a profit, the revenue (income) must be greater than the cost (expenses).

$40\,N > 200 + 10\,N$

$N > \frac{200}{30} = 6\frac{2}{3}$, but N must be a whole number; therefore, N = 7.

6. (9.18) One can either see this as population growth from P in 1981:
P + .03P in 1982
(P + .03P) + 0.6(P + .03P) in 1983
or 1.0918P in 1983, which shows an increase of 9.18%.

Alternate solution:
This can also be seen as an application of successive percents (see sample problem 28).
Here we merely multiply 1.03 x 1.06 = 1.0918, which indicates an increase of 9.18%.

7. $\left(\frac{1}{16}\right)$ The length of the child's head at birth is $\frac{1}{3} \times 18" = 6"$. The length of her body at birth is $\frac{1}{4} \times (18" + 8") = 6\frac{1}{2}"$. The head grew $\frac{1}{2}"$, whereas the body grew 8". The requested ratio is $\frac{\frac{1}{2}}{8} = \frac{1}{16}$.

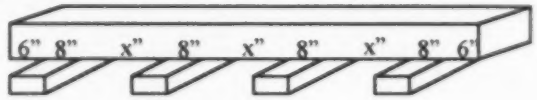
8. (2250) We inspect how many ways each place can be filled. In the ID number the first place can be filled in any of 25 ways (the letter "O" cannot be used). The next place can be filled in 10 ways, and since one of these digits cannot be repeated, the last place can be filled in 9 ways. Therefore, there are 25 x 10 x 9 = 2250 possible ID numbers.

9. (93.3) Since ∠EPB and ∠APD are vertical angles, they are equal. If m∠EPB = 3x, and m∠CPB = x, then m∠EPC = 2x. Therefore, 3x + 40 = 180

$$x = 46\frac{2}{3}$$

$m\angle CPE = 2x = 93\frac{1}{3}$ or $93.3\overline{3}$

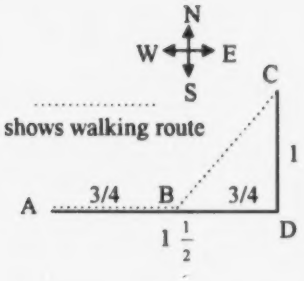
10. ($\frac{28}{3}$ **or 9.33)** The four cinder blocks are placed as shown:



$72 - 2(6) - 4(8) = 3x$

$28 = 3x$

$\frac{28}{3} = x$ or $x = 9.3\overline{3}$.



11. (2) The problem here is to find BC. Using the Pythagorean theorem in ΔBDC, $\left(\frac{3}{4}\right)^2 + 1^2 = y^2$, $y = 1\frac{1}{4}$ (you can use a calculator!).

Alternative solution: You may have recognized ΔBDC as one similar to a 3,4,5 triangle. Use it to find BC.

That is, $BD = \frac{3}{4}$, $CD = \frac{4}{4}$. Therefore, $CB = \frac{5}{4}$.

(N.B. we have $\frac{1}{4}$ of a 3-4-5 right triangle).

The walk is $\frac{3}{4} + 1\frac{1}{4} = 2$.

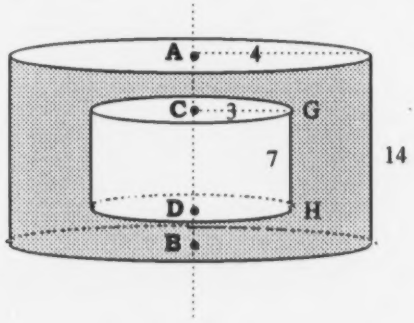
12. (5) We seek to find the smallest number n so that 15m = 18n, or we need to find the lowest common multiple of 15 and 18. To best do this, look at these numbers in prime-factored form.

$15 = 5 \bullet 3$

$18 = 2 \bullet 3 \bullet 3$

If the factors 2 and 3 were added to those of 15 and the factor 5 was added to those of 18, then the two products would be equal. Thus $5 \bullet 18 = 6 \bullet 15$, so the 18" wheel would have to revolve 5 times. Notice we used the radii rather that the circumferences. Since the latter would have involved π and the circumferences are proportional to the radii, the replacement is acceptable.

13. (506) To find the volume of the shaded region, we must take the volume of the smaller cylinder from that of the larger cylinder.



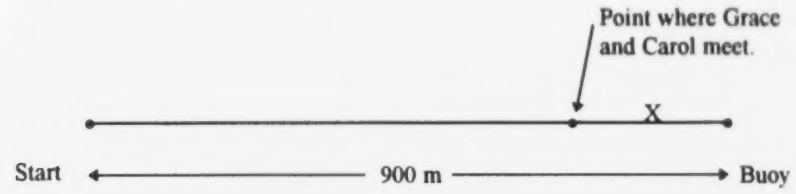
That is, $\pi(4)^2(14) - \pi(3)^2(7)$

$\frac{22}{7}(16)(14) - \frac{22}{7}(9)(7)$

Reduce and factor: $22(32 - 9) = (22)(23) = 506$

14. (90) The palindromes we seek are 3 digit numbers. The first and last place can be filled in any one of 9 ways (the digits 1 - 9). The middle place can be filled in any one of 10 ways (digits 0 - 9). Thus the number of 3-digit numbers is 9 x 10 = 90.

15. (720) If Grace is swimming at $\frac{2}{3}$ Carol's rate of speed, then she covers $\frac{2}{3}$ of Carol's distance in the same time.



Grace's distance = 900 - x
Carol's distance = 900 + x

$$\frac{2}{3}(900 + x) = 900 - x$$

$$x = 180$$

Grace's distance = 900 -180 = 720

TEST 22 SOLUTIONS

1. $(\frac{1}{3})$ $\quad 3^{3x+1} = (3^3)^{2x}$

$$3^{3x+1} = 3^{6x}$$

Therefore, $3x + 1 = 6x$

$$x = \frac{1}{3}.$$

2. $(\frac{4}{3}$ or $1.\overline{33})$ To find the slope: $\frac{\Delta y}{\Delta x} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{-4 - 0}{-3 - 0} = \frac{-4}{-3}$.

3. $(\frac{9}{4}$ **or 2.25)** $\frac{1}{2} \cdot 2 = \left(\frac{1}{2}\right)^2 + 2 = \frac{1}{4} + 2 = \frac{9}{4}$ or 2.25.

4. (2) The traditional way to do this problem is to recall the relationship: Rate × time = distance. The distance traveled by one car is 40t and by the other car is 60t. Since together they travel a total distance of 200 miles:

$$40t + 60t = 200$$

$$t = 2$$

5. (24) The two papers will appear on the same day every 15 days (3 × 5). In 365 days there are 24 times that can happen. $\left(\frac{365}{15} = 24\frac{1}{3}\right)$.

6. (0) The term $(-3)^{99}$ refers to the product of 99 factors of -3. This is a negative product, that is, $-(3)^{99}$. Thus $3^{99} - 3^{99} = 0$.

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7. (7,542) To find the volume of the silo, we use the formula: $V = \pi r^2 h$.

$$V = \frac{22}{7}(10)^2(30)$$

$$V = \frac{22}{7}(3000)$$

$V = 9428.571429$ cubic feet (using a calculator is convenient)

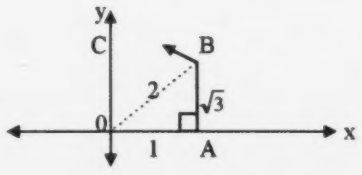
Since one bushel is about 1.25 cubic feet, the number of bushels required $= \frac{9428.57}{1.25} \approx 7542.857$.

There are then 7542 full bushels.

8. (125) In isosceles ΔBCE, the base angles have the same measure $(55)^0$. Since BE = BC and $\overline{BC}$ = AD, BE = AD, and trapezoid ABED is isosceles. Therefore, $\angle E \cong \angle D$. Since $m\angle D = 55^0$, and $\overline{AB} \parallel \overline{DE}$, $m\angle DAB = 125$.

9. (19) In order for there to be a winner, there need to be 19 losers. To have 19 losers we need 19 games.

10. (2) The only crucial point is the center of the circle $(1, \sqrt{3})$ and its movement. ΔAOB is a 30-60-90 triangle, hence $m\angle AOB = 60^0$, leaving 30^0 for $\angle BOC$. When B rotates through this angle it lands on the y-axis, a distance $OB \leq 2$ from 0. The new coordinates of the center of this circle are (0,2).



11. (19) For each automobile accident (a) there were 4 motorcycle accidents (m). For every 2 automobile accidents there were 8 motorcycle accidents. For every 8 motorcycle accidents there was a truck accident (t).

Thus $4m = a$

$8m = 2a$

$8m = t$; therefore, $2a = t$

The ratio of accidents $t : a : m = 1 : 2 : 8$

For the given town $1x + 2x + 8x = 55$

$x = 5$

The number of automobile accidents $2x = 10$.

12. (51.3) The distance traveled by David is $\left(\frac{28}{60}\right)(33)$km, which is equal to the distance traveled by the cyclist $\left(\frac{18}{60}\right)$r, where r is his speed. Therefore, $\left(\frac{28}{60}\right)(33) = \left(\frac{18}{60}\right)r$ and $r = 51\frac{1}{3}$ or 51.3.

13. (82) Creating a chart is a great help in organizing the data given.

age	18	30	42	54	66	78
life expect.	67	68	70	73	77	82

We notice that there is a constant difference of 12 between the entries in the top row, whereas the difference increases by 1 for each succeeding entry in the bottom row. Continuing this pattern gives us a life expectancy of 82 for 78-year-olds.

14. (120) It is very easy to get bogged down with a problem like this. Instead of trying to find the water content of each day's mixture, simply ask the key question: "How much *water* has been added to the mixture each day?" This key question asks for the "complementary substance" to the one required.

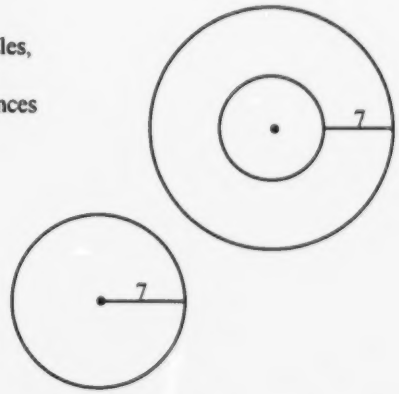
On the first day 1 oz of water is added.
On the second day, 2 oz of water is added.
On the third day, 3 oz of water is added.
On the fifteenth day, 15 oz of water is added.
Why was there no water added on the 16^{th} day?

Therefore, 1+2+3+4+5+6+7+8+9+10+11+12+13+14+15=120 ounces of water that have been consumed.

An **alternate solution** would involve asking the question: "How much has he drunk, in total?"
Then deduct the alcohol (16 oz).
Thus $1 + 2 + 3 + 4 + \ldots + 15 + 16 - 16 = 120$.

15. (44) Method 1: Since we do not know the sizes of the circles, we can make the inner circle very, very small. So small that it shrinks to a point. Then the difference between the circumferences is the circumference of the outer circle: 0.

That is, $C = 2\pi r = 2 \cdot \left(\frac{22}{7}\right) \cdot 7 = 44$



Method 2:
Let r = radius of inner circle
$r + 7$ = radius of outer circle
Then $C - c = 2\pi(r + 7) - 2\pi r$
$C - c = 2\pi r + (2\pi)(7) - 2\pi r$
$C - c = 2\left(\frac{22}{7}\right)(7) = 44$

VI. Index to Problems by Topic

The following index classifies all the problems included in this book according to subject matter. As is often the case in mathematics, a problem may be categorized in more than one way. Therefore, in many instances a problem has been listed in more than one category. As a result, the reader may find a wide diversification of problems in most of the categories listed, yet each entry will be directly related to the topic heading. This index will be useful in providing intensive drill or review of a particular topic. It is best, however, to take the practice tests as a whole before using them to review specific areas in this way.

Symbols used:
II:8 refers to Part II (Sample Problems), question 8.
III:8 refers to Part III (Sample Quantitative Comparisons), question 8.
14:8 refers to Practice Test 14, question 8 (Part IV).

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ANSWER KEY

Mathematics Aptitude Tests

Detailed solutions for all problems in the practice tests of Part IV begin on page 142.

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1. B	1. E	1. C	1. A	1. C	1. C	1. E	1. C
2. D	2. E	2. E	2. E	2. A	2. B	2. A	2. E
3. E	3. B	3. B	3. D	3. B	3. B	3. A	3. E
4. E	4. B	4. E	4. B	4. B	4. A	4. E	4. B
5. A	5. D	5. D	5. B	5. A	5. C	5. C	5. A
6. A	6. C	6. E	6. C	6. D	6. E	6. C	6. D
7. C	7. D	7. E	7. D	7. D	7. E	7. B	7. B
8. C	8. A	8. E	8. B	8. C	8. E	8. D	8. C
9. D	9. E	9. D	9. D	9. B	9. E	9. C	9. B
10. D	10. B	10. B	10. D	10. E	10. E	10. A	10. B
11. D	11. E	11. C	11. C	11. B	11. B	11. E	11. D
12. C	12. D	12. D	12. E	12. E	12. C	12. C	12. D
13. A	13. C	13. B	13. E	13. D	13. E	13. A	13. A
14. B	14. D	14. A	14. A	14. A	14. E	14. D	14. C
15. A	15. A	15. E	15. B	15. C	15. D	15. D	15. B
16. B	16. A	16. A	16. A	16. C	16. B	16. B	16. C
17. C	17. A	17. B	17. C	17. B	17. C	17. B	17. A
18. C	18. C	18. B	18. C	18. B	18. B	18. B	18. C
19. E	19. C	19. C	19. B	19. C	19. A	19. A	19. C
20. B	20. C	20. B	20. C	20. E	20. D	20. B	20. D

Test 2	Test 4	Test 6	Test 8	Test 10	Test 12	Test 14	Test 16
1. B	1. C	1. C	1. A	1. B	1. D	1. C	1. C
2. C	2. C	2. B	2. B	2. D	2. A	2. D	2. A
3. A	3. B	3. C	3. C	3. D	3. B	3. C	3. B
4. B	4. D	4. A	4. B	4. D	4. E	4. E	4. C
5. B	5. D	5. A	5. D	5. C	5. D	5. D	5. E
6. E	6. C	6. C	6. E	6. E	6. A	6. E	6. C
7. A	7. D	7. A	7. D	7. E	7. A	7. A	7. C
8. D	8. E	8. C	8. D	8. D	8. C	8. C	8. D
9. C	9. C	9. A	9. B	9. A	9. C	9. D	9. C
10. B	10. A	10. A	10. D	10. B	10. B	10. C	10. A
11. A	11. B	11. C	11. D	11. C	11. C	11. B	11. B
12. B	12. E	12. D	12. C	12. D	12. A	12. D	12. A
13. E	13. C	13. C	13. C	13. E	13. D	13. E	13. E
14. E	14. D	14. C	14. B	14. A	14. D	14. C	14. C
15. D	15. C	15. B	15. B	15. B	15. C	15. B	15. E
16. B	16. C	16. B	16. E	16. C	16. C	16. D	16. B
17. E	17. D	17. D	17. D	17. E	17. B	17. D	17. C
18. E	18. B	18. D	18. C	18. A	18. E	18. D	18. B
19. E	19. C	19. C	19. E	19. D	19. D	19. C	19. A
20. A	20. B	20. A	20. C	20. D	20. B	20. D	20. E

Test 17

1. C
2. B
3. A
4. B
5. D
6. A
7. B
8. A
9. A
10. B
11. A
12. B
13. A
14. A
15. A
16. B
17. C
18. B
19. D
20. C
21. A
22. B
23. C
24. B
25. B
26. D
27. B
28. C
29. A
30. A
31. C
32. B
33. D

Test 18

1. A
2. B
3. A
4. A
5. B
6. A
7. B
8. D
9. B
10. B
11. D
12. D
13. B
14. D
15. A
16. C
17. A
18. A
19. A
20. C
21. C
22. A
23. B
24. C
25. B
26. D
27. D
28. A
29. C
30. A
31. C
32. A
33. B

Test 19

1. B
2. A
3. B
4. D
5. A
6. C
7. C
8. B
9. A
10. B
11. B
12. B
13. B
14. C
15. D
16. B
17. C
18. D
19. D
20. C
21. A
22. A
23. B
24. C
25. C
26. D
27. D
28. A
29. B
30. B
31. B
32. B
33. A

Test 20

1. D
2. A
3. A
4. B
5. D
6. D
7. D
8. C
9. B
10. A
11. A
12. B
13. A
14. D
15. C
16. D
17. D
18. D
19. C
20. B
21. A
22. D
23. A
24. B
25. B
26. D
27. B
28. A
29. B
30. B
31. A
32. A
33. A

Test 21

1. $\frac{7}{11}$ or .636
2. 6
3. $\frac{40}{3}$ or $13.\overline{3}$
4. 96
5. 7
6. 9.18
7. $\frac{1}{16}$
8. 2250
9. $93.\overline{3}$
10. $\frac{28}{3}$ or $9.\overline{3}$
11. 2
12. 5
13. 506
14. 90
15. 720

Test 22

1. $\frac{1}{3}$
2. $\frac{4}{3}$ or $1.\overline{33}$
3. $\frac{9}{4}$ or 2.25
4. 2
5. 24
6. 0
7. 7,542
8. 125
9. 19
10. 2
11. 10
12. $51.\overline{3}$
13. 82
14. 120
15. 44

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